A Bond Graph Model for the Extensor Mechanism of Human Finger

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Abstract

A musculoskeletal Model of the extensor mechanism of human finger is presented in this paper using Bond Graph. Dynamics of joints is analyzed. The bones are considered as rigid links \([1-3]\). The muscle action is modeled using a string-tube configuration, where each muscle fiber is considered as a flexible string-tube. Actuation of phalanges is achieved by network of tendons called Winslow’s Rhombus. Extension or contraction of the muscle fibers result in relative motion, a combination of translation and rotation, between the mating bones. Each of the component subsystems such as bones, muscles, rotational couplings, etc. have been modeled as Word Bond Graph Objects. The integration of rigid skeletal bone subsystem and the soft muscle subsystem is especially simplified using the unified approach of Bond graphs. Nonlinear stiffness is modeled between phalanges and the tendons so that tendon fibers remain at predefined distance from the bone surface to avoid both pinching as well as detachment. This model will be helpful in understanding the functioning of extensor mechanism, effect of different loading conditions and tension distribution among different members. It can further act as a computational platform to simulate the extensor model under different set of conditions and study relationships among different parameters. Apart from this many models over simplify the properties of the tendinous network for the sake of simplification of model and ease in simulation. This limitation has been overcome in Bond Graph based model.

Keywords: Bond graph, musculoskeletal actuation, modeling and simulation, Tendons

1 Introduction

Models of biomechanical systems are crucial to understand the working of human body with a new perspective. Modeling of musculoskeletal biomechanical systems poses a challenging task due to peculiarities of these systems. Muscles exhibit nonlinear time dependent viscoelastic properties. Joints are not rigidly constrained due to presence of articular cartilage. Cartilage provides very low friction movement in joints as compared to man made anti friction materials. To add more complexity, cartilage material exhibits nonlinear viscoelastic properties. In the musculoskeletal system, the function of the tendons is to transmit forces from muscles to bones in order to move the bones, whereas that of ligaments is to join bones together to form joints. Tendons graze along the bone while imparting effort from the muscle to the bone. They maintain proximity to the bone while providing rotation to the corresponding joint. Proper excursion and gliding of the tendon determine the efficiency of tendon in transmitting the muscle forces to the skeletal system \([4]\), \([5]\).

Computer simulation requires that intricacies of biomechanical systems are precisely captured and translated into computer code that represents a numerically solvable mathematical model of which the solution in the form of time trajectory of the states and consequently of all variables that depend on these states can be numerically approximated. Numerical simulation on digital computer aids in understanding the dynamical behavior of such complex systems.

Bond graph is one of the most powerful and versatile problem solving tools for dynamic systems and has been employed here for the modeling of this elaborate biomechanical system \([6]\), \([7]\), \([8]\). Bond graph is a domain-independent graphical description of dynamic behavior of physical systems (e.g. electrical, mechanical, hydraulic, acoustical, thermodynamic, and material). It is based on transactions of power and depicts causality of physical systems very elegantly. Derivation of system equations and coding for simulation can be done directly and algorithmically from the Bond graph.

Non-linear stiffness is introduced to account for presence of soft cartilage in joints. Similarly non-linear stiffness is introduced between fixed points on bones and floating points. It acts as a soft spring a little away from the vicinity of phalange surface but becomes extremely stiff when it approaches closer to it \([9]\), \([10]\).

1.1 Extensor mechanism of Hand
The anatomy of the hand is efficiently organized to carry out a variety of complex tasks. These tasks require a combination of intricate movements and finely controlled force production. The shape of the bony anatomy in conjunction with the arrangement of soft tissues contributes to the complex kinesiology of the hand [11]. Human finger, in general has three bones called phalanges. These are connected by three joints to the palm (DIP, PIP, and MCP). Joints have unique characteristics to provided compliant motion in different directions as shown in Fig. (1).

The function of tendons is to transmit forces from muscles to bones in order to actuate joints. The tendon lubrication system reduces the friction during tendon excursion. In many cases tendons cross more then one joint and impart rotation to these joints. Tendons graze along the bone while imparting effort from the muscle to the bone. They maintain proximity to the bone while providing rotation to the corresponding joint. Proper excursion and gliding of the tendon determine the efficiency of tendon in transmitting the muscle forces to the skeletal system [12], [13].

A computational environment is proposed to describe and simulate 3D biomechanical systems. The extensor mechanism of the fingers, contain complex tendinous networks and were not previously amenable to biomechanical simulation. Two different topologies of the extensor mechanism were simulated and observed that all other things being equal, moderate changes in the topology of the extensor mechanism greatly affect the distribution of load through its individual elements, and will affect the biomechanical predictions of finger motion and force [14].

Available literature deals with the kinematics and dynamics of human hand or finger systems [15-18]. The models were based on Lagrangian formulations. Modeling complexities such as joint and tendon friction, and interaction of soft muscles with rigid skeletal dynamics are difficult using this approach, and require simplifying assumptions. Experimental studies of finger forces for various patterns of muscle excitation have been reported in [19-21].

A bond graph model of a joint comprising of two mating bones, actuated by a pair of muscle fibers on opposite sides, was presented [9]. Three dimensional motion of the bones was considered. The effect of actuation of flexible muscle fiber along the rigid bones was captured and demonstrated using simulation of the bond graph model.

1.2 Word Bond Graph Objects (WBGO)

Sub systems can be modeled as objects and can be reused conveniently. This leads to an object oriented approach of physical system modeling. The Word Bond Graph Object (WBGO) can be prepared for each subsystem and assembled subsequently to obtain the complete model of the system. Every WBGO has well defined input and output handles to interact with the other WBGOs and the system as a whole. This modular approach enables rapid modeling in case of similar repeated subsystems. It imparts modularity and flexibility to the modeling process as any number of WBGOs, for muscle fibers, bones, joints etc., can be added, deleted or altered rapidly and easily.

Bond Graph model of extensor mechanism is divided into WBGOs, e.g. phalanges, rotational couplings, translational couplings, tendons, etc [22].

2 Bond Graph Model

The integration of rigid skeletal bone subsystem and the soft muscle subsystem is especially simplified using the unified approach of Bond graphs [6-8]. Each of these repeated subsystems, i.e. bones/digits, couplings, tendons, etc. are modeled and used as a WBGOs. The conceptual arrangement is depicted in Fig. (2).

2.1 Translation of digits

Translation of bone is explained by taking example of the distal phalange as it has less number of attachments. $\dot{\mathbf{p}}_3$ is translational velocity of centre of mass of upper bone observed and expressed in the inertial frame $\theta$. Translational momentum $\dot{\mathbf{p}}_3$ is observed and expressed in inertial frame $\theta$, considering the mass of bone 3 to be concentrated at its centre of mass $C_3$. 

![Diagram of Human Finger](image)
Application of forces on the bone due to actuation of muscles results in change of its linear momentum [1-3].

\[
\frac{d}{dt} \mathbf{p}_3 = \sum \mathbf{F}_i
\]  

The \(1_{c_{3}}\) junction sums up the forces acting on different points on bone 3 and provides resultant effort which changes the translational momentum \(\mathbf{p}_3\) of bone 3.

\[
\begin{bmatrix}
\mathbf{c}_1 \mathbf{F}_{c_{3}}
\end{bmatrix}
\]  

is a modulated multibond transformer and represents a skew symmetric matrix obtained from vector \(\mathbf{c}_1 \mathbf{F}_{c_{3}}\) which represents position vector of point \(O_{c_{3}}\) on the bone 3 at the joint with respect to the centre of mass \(C_{3}\) of the bone, observed and expressed in frame 0.

Matrix \(\begin{bmatrix}
\mathbf{c}_1 \mathbf{F}_{c_{3}}
\end{bmatrix}\) keeps on changing as the bone undergoes a change in orientation.

**2.2 Rotation of digits**

\(1_{\omega}\) represents the rotational part of motion of upper bone and sums up the efforts as follows

\[
\begin{bmatrix}
\omega
\end{bmatrix} \mathbf{p}_3 = e_{d_{3,10}} = e_{D_{1,3}^{3,4}} - (e_{d_{3,3}} + e_{d_{3,4}} + e_{d_{3,7}})
\]  

Where \(\begin{bmatrix}
\omega
\end{bmatrix} \mathbf{p}_3\) is angular momentum of the bone 3. \(\begin{bmatrix}
\omega
\end{bmatrix}\) represents the Inertia tensor of the bone 3 about its center of mass, expressed in frame 0.

Orientation of each frame can be expressed with respect to inertial frame 0 with the help of corresponding rotational transformation matrices. \(\begin{bmatrix}
\omega
\end{bmatrix} \mathbf{R}\) represents the rotational transformation matrix from frame 3 to inertial frame 0. Its variation with time due to change of orientation is given by

\[
\begin{bmatrix}
\omega
\end{bmatrix} \mathbf{R} = \begin{bmatrix}
\omega
\end{bmatrix} \mathbf{R}_0 \begin{bmatrix}
\omega
\end{bmatrix}
\]  

Initial value of \(\begin{bmatrix}
\omega
\end{bmatrix} \mathbf{R}\) is obtained from initial orientation of the bone 3 with respect to inertial frame 0.

**2.3 Joints**

Joints between the bones are modeled as a revolute joint with viscoelastic behavior due to the presence of soft cartilage [24]. Viscoelastic behavior is modeled by providing C and R elements between the \(0\) junctions of all the bones (between \(O_{12}\) and \(O_{21}, O_{22}\) and \(O_{31}\)). Nature of elements C and R depend on the properties of the biological material.

Similarly rotational motion is allowed about one axis only and limited motion is permitted about the other two axes.

\[
\epsilon = \mathbf{R}_0 \mathbf{R}_3
\]  

where \(\mathbf{R}_0\) is relative velocity between the bone 3 and bone 2, expressed in frame 0.

**2.4 Tendinuous Network (Winslow’s Rhombus)**

The tendon network topology of the Winslow’s rhombus is shown in Fig. (5). Two types of points are defined. Fixed points (\(D_{3,1}, D_{2t}, D_{1t}, \ldots\)) are defined on the surface of bones and any effort acting on these points will result in change in position or orientation of bones. Floating points (\(P_{2t}, P_{1t}, P_{1f}, \ldots\)) on tendons have restricted movement as they are required to capture gra-
ing movement of tendons along bones. These points are attached to each other through tendon segments. Floating points are attached to fixed points through non-linear spring \( (C_{N10} \text{ at bond NL21}_1\_3) \) which allows some movement of floating point but prevents it from penetrating the bone. Connections between \( D_{31} \text{-} P_{21} \) and \( D_{23} \text{-} P_{1} \) are in the form of string-tube mechanism [9], [23].

Jerk less or smooth motion may be imparted to the network though \( P_{16}, P_{15}, P_{16} \). This is applied using sources of flow \( S_j \) which are connected to the motion of these points through viscoelastic couplings, provided by \( C \) and \( R \) elements. Tendons attached to these points further pull other floating points, which in turn apply effort to fixed points through non-linear spring and change orientation of bones.

\( C_{TN1}_7 \) and \( R_{TN1}_6 \) are responsible for the material properties of tendons and \( R_{TN1}_3, R_{TN1}_9 \) accounts for friction losses during the movement of tendon.

### 2.5 Alternate Model

The floating point on the join of tendons receives forces from different directions depending on movement of connecting tendons. The point has also to graze along the bone surface, and not penetrate it. The approach to model this feature is discussed here. For a connection of three tendons, we have three hooks, on the bone surface, through which respective tendons pass, Fig. (6). The location of these hook points is fixed on the bone, but that of the floating point changes due to the application of tendon forces. The unit directions between the hook points and the floating node are simply computed from the motion of the floating point with respect to the hook points. These unit vectors provide the requisite moduli for respective MTFs which convert the directional forces to tensions in tendons. The tensions in tendons are transmitted, through the string-tube model, to the termination points of tendon on bone and other floating points as may be the case. Hooks can be provided on all crucial points to keep the tendons in place.

This approach (1) eliminates the necessity of non-linear springs which cause computational difficulties, (2) retains the floating points on the bone surface, and (3) handles the issue of tendon connections along different changing directions.

### 5 Conclusions

Musculoskeletal dynamics of the extensor mechanism of human finger is presented in this paper using Bond Graph. Bones have been considered as rigid links and subjected to multibond graph formulation of rigid body dynamics. Dynamics of joints has been accounted using suitable viscoelastic constraint relaxation. The muscle action is modeled using a string-tube configuration, where each muscle fiber is considered as a flexible string-tube. Actuation of phalanges is achieved by network of tendons called Winslow’s Rhombus. Extension or contraction of the muscle fibers result in relative motion, a combination of translation and rotation, between the mating bones. Each of the component subsystems such as bones, muscles, rotational couplings, etc. have been modeled as Word Bond Graph Objects. The integration of rigid skeletal bone subsystem and the soft muscle subsystem is especially simplified using the unified approach of Bond graphs. Nonlinear stiffness is modeled between phalanges and the tendons so that tendon fibers remain at predefined distance from the bone surface to avoid both pinching as well as detachment. This creates difficulties for numerical integration. This difficulty has been overcome through an approach for modeling tendon connections at floating points using hook points on bones. This model will be helpful in understanding the functioning of extensor mechanism, effect of different loading conditions and tension distribution among different members. It can further act as a computational platform to simulate the extensor model under different set of conditions and study relationships among different parameters.

### 6 Nomenclature

- \( C_i \) = Center of mass of bone \( i \).
- \( \mathbf{I}_i \) = Inertia tensor of bone \( i \) with respect to its center of mass, expressed in inertial frame \( \Theta_i \in \mathbb{R}^{3 \times 3} \).
- \( K_i \) = Stiffness element attached to bond \( i \); \( \in \mathbb{R}^1 \).
- \( \mathbf{sP}_i \) = Translational momentum of bone \( i \) observed and expressed in the inertial frame \( \Theta; \in \mathbb{R}^3 \).
- \( \mathbf{cP}_i \) = Angular momentum vector of bone \( i \) with respect to its center of mass and expressed in inertial frame \( \Theta_i \in \mathbb{R}^3 \).
- \( R_i \) = Damping element attached to bond \( i \); \( \in \mathbb{R}^1 \).
- \( \mathbf{cR}_i \) = Rotation matrix describing orientation of frame \( i \) with respect to inertial frame \( \Theta \).
\[ 0 \dot{R} = \text{Time derivative of } 0 R \in \mathbb{R}^{3 \times 1} \]

\[ \dot{0} \gamma = \text{Velocity of center of mass of the bone } i \text{ observed and expressed in inertial frame } \theta_i \in \mathbb{R}^{3 \times 1} \]

\[ \dot{S} = \text{Speed of string end at } P ; \in \mathbb{R}^{3 \times 1} \]

\[ \begin{bmatrix} \dot{0} \gamma \end{bmatrix} = \text{Skew symmetric cross product matrix obtained from vector } 0 \gamma \in \mathbb{R}^{3 \times 3} \]

\[ [U] = \text{Unit matrix; } \in \mathbb{R}^{3 \times 3} \]

\[ \dot{\omega} \in \text{Angular velocity vector of frame } i \text{ observed and expressed in the inertial frame } \theta_i \in \mathbb{R}^{3 \times 1} \]

References


Fig. 5: Multibond graph model for the musculoskeletal dynamics of the finger actuated by the tendinous network of the Winslow’s rhombus. The system showing phalanges and the tendon network is also shown on the upper right.