A *Compliant Mechanism Kit* with Flexible Beams and Connectors

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Abstract

We present the concept, prototypes, and an optimal design method for a *compliant mechanism kit* as a parallel to the kits available for rigid-body mechanisms. The kit consists of flexible beams and connectors that can be easily hand-assembled using snap fits. It enables users, using their creativity and mechanics intuition, to quickly realize a compliant mechanism. The mechanisms assembled in this manner accurately capture the essential behavior of the topology, shape, size and material aspects and thereby can lead the way for a real compliant mechanism for practical use. Also described in this paper are the design of the connector to which flexible beams can be added in eight different directions; and prototyping of the spring steel connectors as well as beams using wire-cut electro discharge machining.

It is noted in this paper that the concept of the kit also resolves a discrepancy in the finite element (FE) modeling of beam-based compliant mechanisms. The discrepancy arises when two or more beams are joining at one point and thus leading to increased stiffness. After resolving this discrepancy, this work extends the topology optimization to automatically generate designs that can be assembled with the kit. Thus, the kit and the accompanying analysis and optimal synthesis procedures comprise a self-contained educational as well as a research and pragmatic toolset for compliant mechanisms. The paper also illustrates how human creativity finds new ways of using the kit beyond the original intended use and how it is useful even for a novice to design compliant mechanisms.

Keywords: Compliant mechanism kit, beam finite element analysis, topology optimization

1 Introduction

Compliant mechanisms are the joint-less mechanisms that transmit or transform the motion or force due to elastic deformation rather than through hinges and sliders as in their rigid linked counterparts. The primary advantages of compliant mechanisms are fewer parts, fewer assembly steps, absence of backlash and obviating the need for lubrication. The absence of hinges makes compliant mechanisms attractive for many applications [1, 2] including the emerging areas of micro and nano scale systems [3]. In spite of their many advantages, compliant mechanisms are not yet widely used nor are they taught widely in undergraduate engineering courses. A reason for this slow, but gradually increasing, adoption of compliant mechanisms may be that designing them is a bit involved because one has to deal with elastic deformations—often geometrically nonlinear. Designing or even analyzing compliant mechanisms usually requires access to finite element analysis (FEA) software. The pre- and post-processing involved in FEA, i.e., drawing or modifying the computer model and meshing it and then visualizing its deformation, makes it difficult for the designer to exercise creativity and intuition. Prototyping a compliant mechanism is another difficulty because one needs to machine it usually using a CNC machine because of their not-so-simple geometry. Contrast this situation with rigid-body mechanisms whose motion is more easily visualized. Or they can be built easily using even cardboard and pins. Furthermore, several kits have been developed as practical toolsets for designing rigid-body mechanisms wherein mechanisms can be realized by simple hand-assembly from the available parts. Developing such a kit for compliant mechanisms is the focus of this work.

There is a second motivation for *developing a compliant mechanisms kit*. A beam finite element is a popular choice for analysis or topology optimization of compliant mechanisms [4], which have certain advantages over continuum finite elements. They are attractive in analysis because most compliant mechanisms comprise slender beam-like segments. The attractive features for topology optimization are fewer elements required and the potential to obtain distributed compliance better than what can be achieved with continuum element-based methods. However, there is an inconsistency in using beam finite elements for modeling compliant mechanisms. It is due to the modeling of the connections where two or more beams of different widths intersect. Consequently, a prototype of an analyzed or designed mechanism is found to be stiffer than its beam FE beam-model. This is illustrated with an example presented next.

Figure 1(a) schematically shows a beam connection as it is assumed in the FE model while Fig. 1(b) shows the same beam connection as it appears in the prototype. To accommodate the finite size of the cutting tool and to reduce stress concentrations, a fillet is indeed unavoidable. But it adds extra material at the joint. It also reduces the actual deformable lengths of the intersecting beams as opposed to the lengths assumed in the FE model. Hence the joint becomes stiffer in the prototype than its FE model [5].

When we use beam ground structure in topology optimization, there are many beam-intersections. Hence, the aforementioned problem becomes even more important. While developing the methods to resolve this issue [6], it was realized that a better way is to simply avoid the intersections of flexible beams by having a semi-rigid connector in analysis, design, and the real prototype. This led to the idea of a *compliant mechanisms kit*.

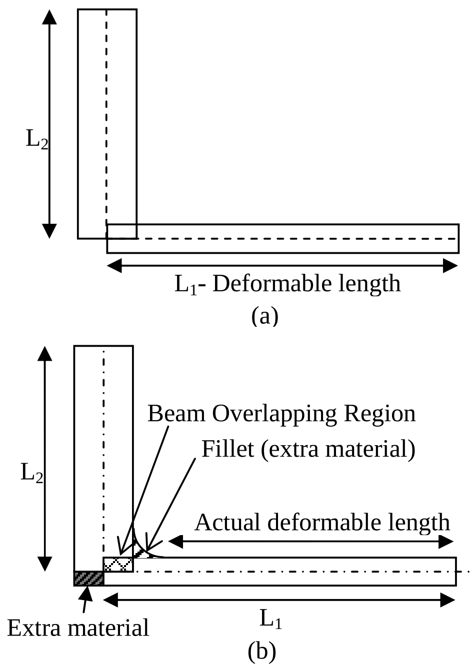


Fig. 1: (a) Ideal beam connection in FE model (b) Actual beam connection in prototype.

The remainder of the paper is organized as follows. We describe the novel concept of the kit and its building blocks in Section 2. The use of the kit is also illustrated in this section. Finite element modeling of a compliant mechanism assembled using the kit is explained in Section 3. Topology optimization, which can generate solutions that are realizable with the kit, is presented in Section 4. Section 5 discusses the uses of the kit and illustrates how creative users can use it in ingenious ways even though they may not be familiar with compliant mechanisms a priori. This point is underscored by the fact that a summer intern, the third author, who has finished only the first year of mechanical engineering undergraduate programme, was able to design novel compliant mechanisms effortlessly.

2 Compliant Mechanisms Kit

Our compliant mechanisms kit consists of two types of building blocks. One is a flexible beam and the second is a semi-rigid connector. The connector undergoes very small to negligible deformation as compared with that of the beams. The connectors are located at the junctions where different beams meet in different directions. Figure 2(a) shows the geometric model of the connector while Fig. 2(b) shows a sample assembly. There are five connectors and eight beams in it. Four beams are longer than the other four beams to make this possible. At this time, we have only two lengths but more could be added. The point we illustrate in this paper is that just with two lengths, many interesting compliant mechanisms can be created. And, two lengths are adequate for a square grid with cross-beams that go diagonally. This is, by the way, the beam ground structure used in topology optimization [7,8].

The geometry of the connector was designed (see Fig. 3) such that a beam can be easily snap-fitted into one of the eight slots. Also seen in the connector are eight shorter crack-shaped notches. These help in inserting a beam by allowing slight deformation of the material on either side of the slot and in restoring the original configuration when the beam is positioned properly. This helps in a tight fit of the beam inside the slot even though it does not take much force for insertion. This is the well-known principle of snap-fits.

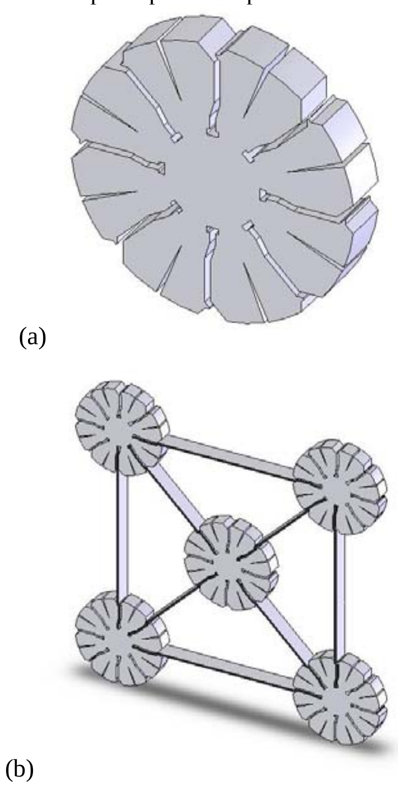


Fig. 2: (a) Semi-rigid connector (b) A simple assembly with five connectors and eight beams.

2.1 Physical building blocks of the kit

We chose spring steel (AISI 1040) for making the physical building blocks. It permits very thin beams that can be easily cut by shearing from a sheet or using wire-cut Electro Discharge Machining (EDM) if precision is desired. The spring steel beams allow reversible large elastic deformation without yielding. The connector, with its intricate and narrow cuts shown in Fig. 3, can also be easily made using spring steel or aluminium using wire-cut EDM.

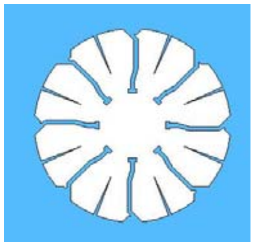
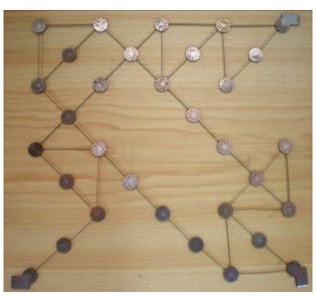


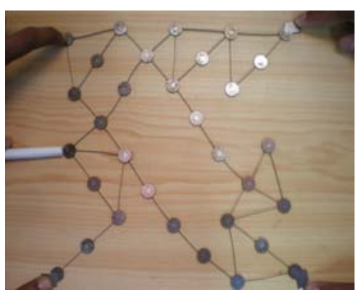
Fig. 3: Top-view of the semi-rigid connector with eight slots for snapping the beams and eight crack-shaped notches for aiding the insertion.

Figure 4(a) shows an assembled compliant mechanism using the physical building blocks. Its four corners are held fixed by gluing it to a base. When we apply a rightward force in the middle of the left edge as shown in Fig. 4(b), the midpoint of the right edge moves to the left. This is a non-intuitive motion. Some amplification of the displacement is also noticeable in the figure.

The horizontal and vertical spacing is 50 mm. The beams are made of spring steel of thickness 0.2 mm and in two standard lengths. The radius of the connector (see Fig. 4c) is 5.5 mm. In all the examples in this paper, the dimensions of beams and connectors remain the same as above.



(a)



(b)

(c)

Fig. 4: (a) A compliant mechanism assembled from the kit (b) The deformed configuration (c) The connector made of spring steel

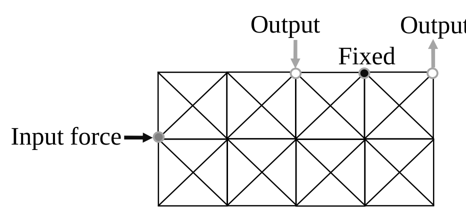
It is well known in the compliant mechanisms literature that the topology is of utmost importance. The connectivity of the beams decides the way the mechanism deforms. As shown in Figs. 4(a-b), particular arrangement of beams leads to non-intuitive motion. This is what a topology optimization algorithm can do by removing the unnecessary beams in a ground structure. The kit makes it easy for a human user to do the same and encourages creativity. Selective removal/addition of the beams can also be done interactively on a computer but a kit gives the feel and realistic geometrically nonlinear motion much more quickly than a nonlinear finite element solution. The topology designed by a human user can be readily used to make a real compliant mechanism for practical use.

Next, we present some examples to illustrate the kinds of compliant mechanisms possible with the kit.

2.2 Examples

Consider three specifications for the design of compliant mechanisms as shown in Figs. 5(a-c). All three have some practical relevance. The first one (Fig. 5(a)) requires that an input force at a point results in an output motion that approximates rotation around a fixed point. This may be useful for disk-drive mechanisms so that they can easily miniaturised with linear microactuators rather than rotary micromotors. The problems of rotary micromotors such as stiction and wear are well known. It would be a good exercise to imagine which beams we should keep in order to get this input-output motion using the compliant mechanism kit.

(a)



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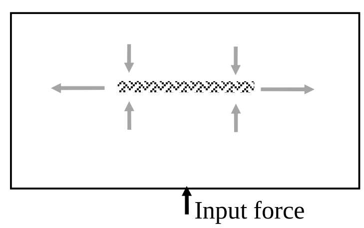

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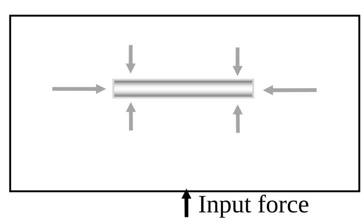
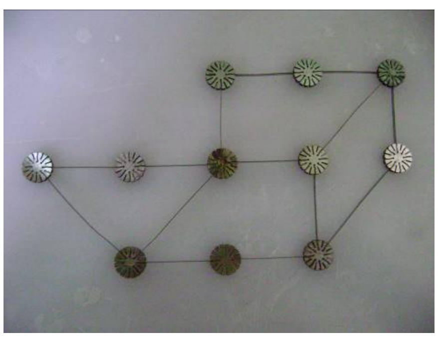


Fig. 5: Required input-output motion specifications for compliant mechanisms (a) An input force giving an approximate rotation of two points about a fixed point (b) grasp and then pull a fibre (c) grasp and then push on a rod.

In Figs. 5(b) and 5(c), we show a different type of specification. In Fig. 5(b), we have a fibre that needs to be grasped with two pairs of points first and then pull it apart. This may be needed to test the strength of a fiber. Note that we can use only a single force to do it because it is convenient and minimizes the need for control. In Fig. 5(c), we have the same problem but here a rod should be grasped first and then pushed in from both the sides. In both the specifications, we do not restrict the reader as to where the compliant mechanism should be fixed. This is in contrast to what is usually done in topology optimization and hence an advantage over topology optimization.

The reader should pause at this time and think of possible solutions to the above three problems. Clearly, there is no one solution to such problems. Here, through the compliant mechanism kit, we are restricting the possible solutions that can be realized quickly. As shown in Fig. 5(a), which beams need to be retained?

Solutions for the three problems are shown in Figs. 6-8. These were created partly with intuition and partly by experimenting with the kit by the third author who had completed the first year of engineering program at the time of doing it. At that level, the concepts of kinematics and strength of materials are still unfamiliar and finite element analysis is unheard of. Yet, the kit made it possible to experiment with the beams and connectors in various ways to develop 'compliant mechanism intuition' so that all three problems (and more) could be systematically thought through to create at least one possible solution for every problem.

(a)


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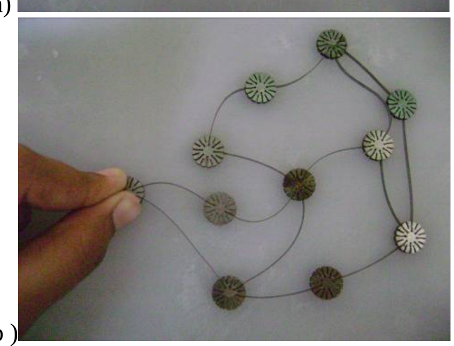
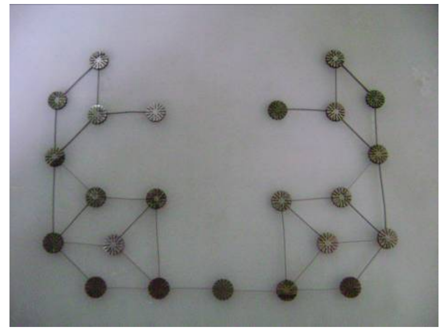


Fig. 6: (a) Solution to problem shown in Fig. 5(a), (b) the deformed configuration wherein a large input force is applied. The desired approximate rotation of two points about a fixed point is noticeable.

(a)


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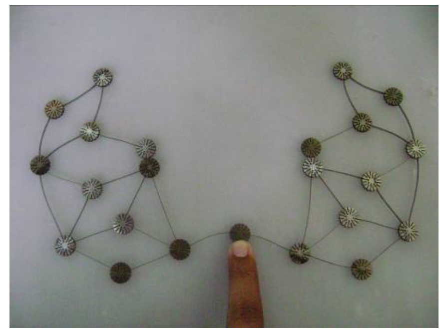
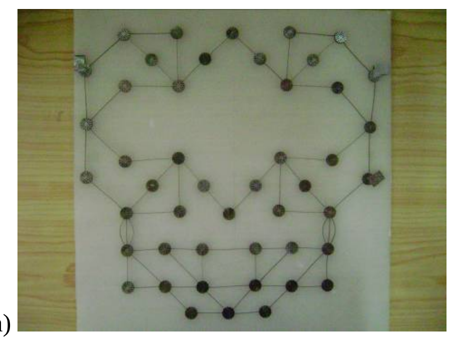


Fig. 7: (a) Solution to problem shown in Fig. 5(b), (b) the deformed configuration. Two pairs of points have come together to grasp and then are pulled apart.

(a)


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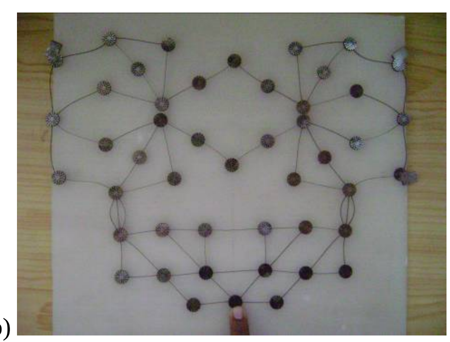


Fig. 8: (a) Solution to problem shown in Fig. 5(c), (b) an intermediate deformed configuration; two pairs of points have come together to grasp. (continued on the next page)

(c) 

Fig. 8: (continued) (c) further application of the input force results in the pushing in of the grasped points.

The deformation, as can be seen in Figs. 6-8, is quite large. Developing an intuition for the large deformations is not easy. Finite element solution, if done in a computer, does not give the result as quickly as the kit can provide. The force we feel at the input is also an added advantage in understanding the behavior of the mechanism.

Also worth noting here is the contact that takes place among different parts of the mechanisms. Such mechanisms are known as contact-aided compliant mechanisms [5]. It enriches the kinematic and stiffness adaptability of compliant mechanisms. Such concepts can also be explored with the help of this kit without having to model the sophisticated mechanics associated with it.

3 FE Modeling of Compliant Mechanisms Made with the Kit

In compliant mechanisms made using the kit, the beams are not directly connected to each other; rather they are connected through the semi-rigid connector. We call it semi-rigid here because it undergoes slight deformation during insertion of the beams. However, once a mechanism is assembled, the connectors are essentially rigid. Therefore, connectors undergo a rigid-body motion. Consequently, we need to appropriately relate the displacements of nodes at which beams are connected to those of the beams' degrees of freedom. This can be done using what are called multi-point constraints in the FEA literature. These constraints are linear for small displacements and nonlinear for large displacements.

In order to analyze these mechanisms, we solve the equations obtained by regular FE assembly and impose the constraint equations. It increases the computation. To avid this, we used a different way of analysis where we introduce a super element in the place of each connector. The stiffness matrix of this element should be large enough so that the element shows a singular mode for translational displacements as well as for small rigid rotations. It is obtained by the influence coefficient method [9]. In this, a unit force is applied at each degree of freedom and the resulting displacements are stored as a column in a matrix. By doing this for all the degrees of freedom, all the columns of the matrix are filled. This matrix is multiplied by a large value to simulate the rigidity of the connector. The connector stiffness matrices are assembled with beam elements and FE equations are solved together.

We note that the problem of joint-stiffening in ordinary beam connections is avoided here. The connector is rigid in both the model and the prototype. Hence, there is no discrepancy as long as the beams obey the assumptions of the Euler-Bernoulli beam theory. In the kit, the beams are made sufficiently slender to ensure this; that is the length is at least 15 times larger than the larger of the cross-section dimensions.

To solve the large deformation problem, we used co-rotational beam elements as implemented in [10]. The load is applied in steps. At every step, the equilibrium equation is solved using the Newton-Raphson algorithm. It requires computing the tangent stiffness matrix and the internal force at each step. It is necessary to separate rigid-body displacements from the total displacements. Figure 9 shows the large displacement of a connector which includes rigid rotation α .The local coordinate system (LCS) makes an angle α with the global coordinate system (GCS). For simplicity, only two nodes are considered, which have coordinates (X_1, Y_1) and (X_2, Y_2) , with respect to origin O . U_L and U_g are displacements in LCS and GCS respectively.

$$\begin{aligned} U_L &= \left[U_x^1 \quad U_y^1 \quad U_\theta^1 \quad U_x^2 \quad U_y^2 \quad U_\theta^2 \right]_L^T \\ U_g &= \left[U_x^1 \quad U_y^1 \quad U_\theta^1 \quad U_x^2 \quad U_y^2 \quad U_\theta^2 \right]_g^T \end{aligned} \tag{1}$$

The mapping between LCS and GCS is given by the transformation:

$$\begin{aligned} \delta U_L &= \lambda \delta U_g \\ F^{\text{int}}{}_L &= \lambda F^{\text{int}}{}_g \end{aligned} \tag{2}$$

where transformation matrix λ is given by

$$\lambda = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 & 0 & 0 & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\alpha) & \sin(\alpha) & 0 \\ 0 & 0 & 0 & -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

U_L is obtained from U_g by subtracting displacements due to rigid rotation of α and mapping them to LCS by transformation matrix λ .

$$U_L = \lambda (U_g - \Gamma) \tag{4}$$

$$\Gamma = \begin{bmatrix} \cos(\alpha)-1 & -\sin(\alpha) & 0 & 0 & 0 & 0 \\ \sin(\alpha) & \cos(\alpha)-1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\alpha)-1 & -\sin(\alpha) & 0 \\ 0 & 0 & 0 & \sin(\alpha) & \cos(\alpha)-1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \\ X_2 \\ Y_2 \\ 1 \end{bmatrix} \tag{5}$$

$$F^{\text{int}}{}_L = K U_L \tag{6}$$

From Eqs. (2), (4), and (6), we can write

$$F^{\text{int}}{}_g = \lambda^T K \lambda \left(U_g - \Gamma \right) \tag{7}$$

Hence, the tangent stiffness matrix is obtained as

$$K_g^{tgt} = \frac{\partial F^{\text{int}}{}_g}{\partial U_g} \tag{8}$$

$$K_g^{tgt} = \lambda^T K \lambda \tag{9}$$

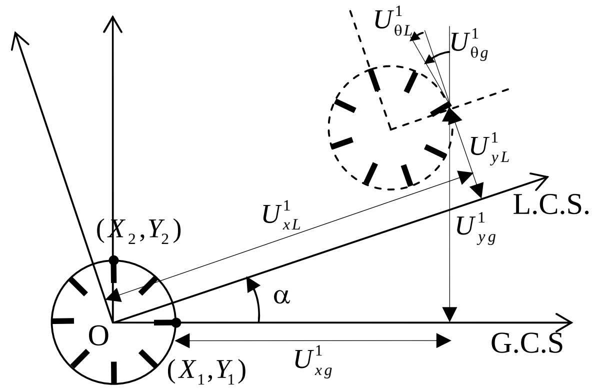


Fig. 10: Solid lines show the original connector and dashed lines show displaced configuration. Mapping is obtained between LCS and GCS.

A test problem is analyzed as shown in Fig. 11, where two beam elements are connected together with a rigid connector. The stiffness of the joint is kept very high so as to make it perfectly rigid. We notice that the joint perfectly undergoes a large rotation.

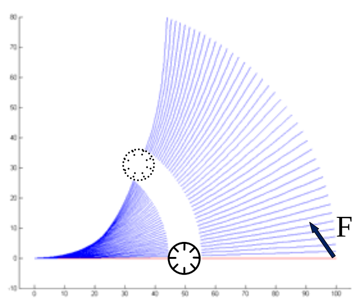


Fig. 10: Test problem of the rigid connector for large deformation problem.

4 Optimal Synthesis by Topology Optimization

Using the FE model as discussed in the previous section, we now proceed to topology optimization that can do what the human user can do with the kit: add connectors and beams as needed to satisfy the specified functionality. The optimization problem is stated as follows.

$$\begin{array}{lll} \underset{x_i}{Maximize} & \dfrac{MSE}{SE} & \\ subject\ to & KU = F & where \\ & KU_d = F_d & MSE = U^T K U_d \\ & V < V^* & SE = \dfrac{1}{2}\ U^T K U \\ & x_i = w\ or\ 0 & \end{array} \tag{10}$$

Here, the ratio of mutual strain energy (MSE) and strain energy (SE) is considered as the objective function to be maximized with respect to widths of the beams in the ground structure grid as the design variables x_i. A spring is provided at the output port where displacement is to be maximized. F_d denotes a unit dummy load applied at the output node in the direction in which the displacement is to be maximized. The volume constraint has V^* as the upper limit on the volume of the beam elements used.

As the designs are intended to be manufactured from the kit, the design variable x_i is constrained to take a value of either 0 or w, where w is standard width of the beams in the kit. So, in fact, it is a binary optimization problem. In order to solve it efficiently using gradient-based optimization, we consider it as a continuous optimization problem, provided that penalization is done on design variable x_i. So, penalization is used where p is penalty factor. Instead of x_i, now X_i is used as beam widths for computing the element stiffness matrices.

$$X_i = w\left(\frac{x_i}{w}\right)^p \tag{11}$$

We solved few benchmark problems using this formulation. The MMA (Method of Moving Asymptotes) algorithm [11] is used for optimization. In all following examples, w is 0.2 mm. Lower bound on x_i is kept at 1e-3 to avoid numerical singularities.

Figures 11(a) and 12(a) show the results of optimization for $p=1$ and $p=2$ respectively for the inverter mechanism wherein the direction of the input force is reversed in the output displacement. A volume constraint of 0.4 is used. The spring at the output has a value equal to 1e-10 times E, the Young's modulus (210 GPa for spring steel). The distribution of beam widths is shown in Figs. 11(b) and 12(b). We notice that widths of almost all the members are pushed to either 0 or w, not only in the case where $p=2$ but also for $p=1$. The reason is attributed to the bending stiffness of beams which is proportional to the cube of their widths. This acts as inherent penalization for beams that are in between 0 or w and are not preferred by the optimization algorithm. Although some of the beams have attained intermediate widths, it is negligible and the mechanism behaves similarly, when those particular beams are deleted. We can also see that with $p=2$, the number of such intermediate elements has decreased.

The mechanism solution assembled using the kit as per Fig. 12(a) is shown in Fig. 13. It is important to note that a human designer might have come up with a different solution as the solution is not unique. Also important to note is that optimization algorithm takes only a few seconds on a desktop computer. This gives the topology optimization an edge. The same is not true when we implement the nonlinear analysis into the topology optimization programme [12]. Those results are not presented in this paper. But it suffices to note that human user with the compliant mechanisms kit can then compete with the computer in coming up with a solution for a desired input-output motion specification.

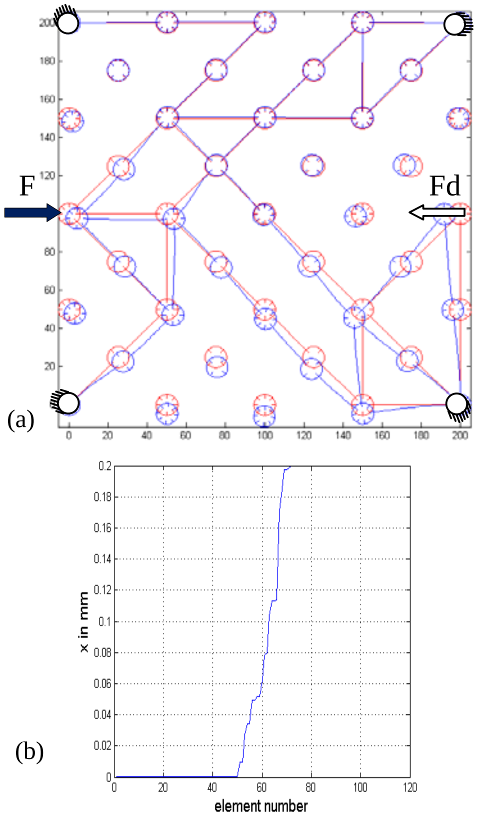


Fig. 11: (a) Inverter mechanism shown with deformed configuration for F = 100 N, (b) Distribution of widths plotted. Displacement amplification is 1.96. $p = 1$.

5 Discussion and Other Uses of the Kit

We showed in Section 2 how the kit can be used to conceive new compliant mechanisms. The one shown in Fig. 8(a) is difficult among the three examples. It is not an easy one to be solved by topology optimization algorithm as it involves to steps (grasp first and then push) and very large deformations as can be seen in Figs. 8(b-c). How was this designed? To see this, consider the actual hand-sketched ideation process shown in Fig. 14. This mechanism was designed in steps following a simple idea of grasping. It was then made sufficiently stiff by experimenting with the kit. And then, the push-part is added in a few steps. Finally, more beams are added to give the mechanism adequate stiffness. The verbatim description of the designer, the third author, is rather long to include here. But is it not unreasonable to assume that many others can also do this to create novel compliant mechanisms.

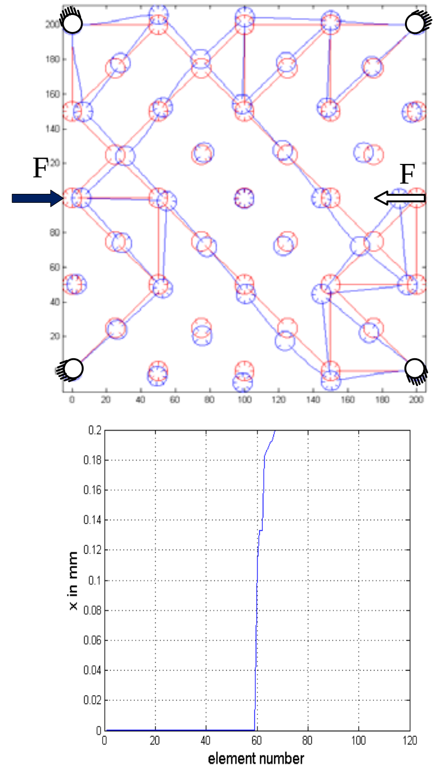
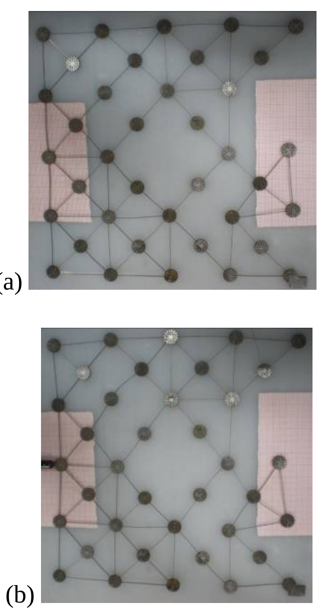


Fig. 12: (a) Inverter mechanism shown with deformed configuration for F = 100 N, (b) Distribution of widths plotted. Displacement amplification is 2.19. $p = 2$.

(a)



(b)

Fig. 13: Compliant mechanism assembled using the kit as per the topology solution shown in Fig. 12(a). (a) Without force, (b) with force.

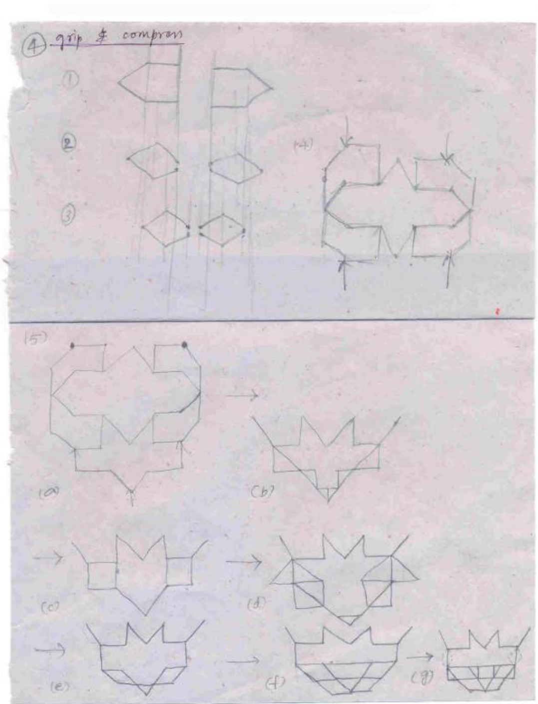
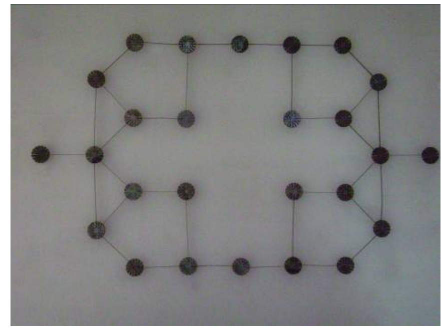


Fig. 14: Hand-sketched thought-process that led to the mechanism assembled with the kit and shown in Fig. 8(a). The thought process is too long to include here but may be partly understood from the sketch.

Human creativity enhances the utility of the kit. Some observations were made when this kit was used by different people. Consider the mechanism shown in Figs. 15(a-b) with its undeformed and deformed configurations. At first sight, it is also a grasp and pull mechanism albeit with two inputs that act on the left and right hand sides. But there is more to it than can be understood at first sight. Observe how the almost rectangular outer profile of the mechanism changes to groundnut (peanut) shape when it is deformed. This is an example of a profile-morphing design. A number of researchers are working on this problem to conceive designs where an aircraft wing can smoothly morph from one shape to another [13]. We find that the kit can be of help in such problems as well. Next, we briefly describe our preliminary attempt at this.

Here, we wanted to conceive an internal arrangement of the beams inside an aerofoil so that its shape can changed with just one input actuation. For this, an user assembled an approximate aerofoil shape with the existing kit. See Fig. 16(a-b). By fixing the upper and lower connectors on the left side and moving the middle connector, the shape can be changed so that the trailing edge can be substantially deflected while the leading edge keeps the shape in comparison. Thus, not unexpectedly, users think of novel ways of using the kit. Using bent beams (see Fig. 8(a-c)) to prevent the buckling or excessive and undesirable deformation of an important beam is another such example.

(a) 

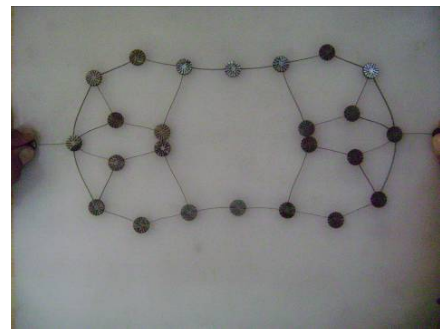
(b) 

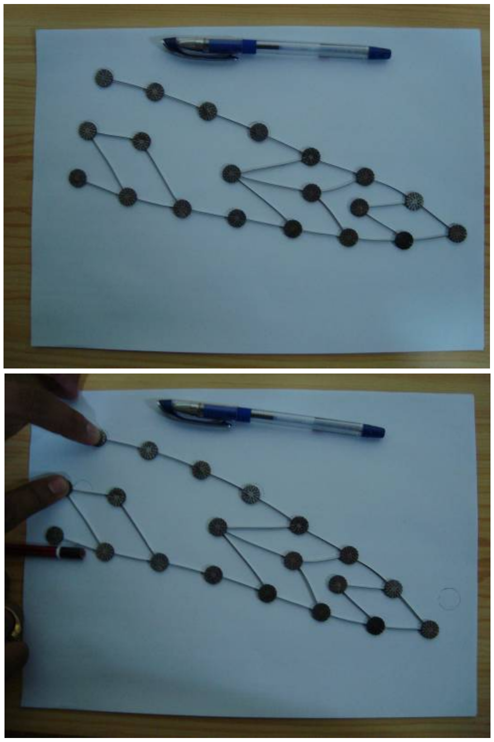
Fig. 15: A mechanism designed for grasping and pulling on an object using two inputs. (a) undeformed, (b) deformed. Notice also how the almost rectangular outer profile of the undeformed mechanism changes to groundnut (peanut) shape in the deformed mechanism. This is a clue to designing morphing topologies using the kit.

6 Conclusions

In this paper, we proposed a kit for making prototypes of compliant mechanisms. The kit comprises a semi-rigid connector and beams of two lengths, all made using spring steel. Using the kit it is possible to realize compliant mechanisms quickly by mere hand-assembly by exercising one's intuition. It also helps in comprehending the results of systematic synthesis process.

We note here that the kit helped circumvent a modeling discrepancy encountered in beam finite element based simulation of compliant mechanisms. Additionally, we presented an efficient way of FEA of the mechanisms made using the kit. We also presented a topology optimization method that takes only a few seconds to give a solution to any arbitrary specifications. We noted that this uses linear modeling and nonlinear modeling, which is not presented here, takes more time so that human user can compete with the optimization algorithm.

The kit's parts along with the optimal design method prove as a pragmatic toolset for design of complaint mechanisms. Such things as contact among the members, buckling, and unspecified fixed locations, etc. can be easily tackled with the kit but not with topology optimization. We also illustrated the other creative ways we can use the kit in designing morphing shapes.



(a)

(b)

Fig. 16: The compliant mechanism kit used for designing morphing wings. (a) An approximate aerofoil shape assembled using the same building blocks—beams of two different lengths and a connector. (b) the changed profile with a single actuation at the middle connector on the left side.

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