Modeling and Simulation of the Dynamics of Crankshaft-Connecting Rod-Piston-Cylinder Mechanism and a Universal Joint Using The Bond Graph Approach

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Abstract

This paper deals with modeling and simulation of the dynamics of two commonly used mechanisms, (1) the Crankshaft – Connecting rod – Piston – Cylinder system, and (2) the Universal Joint system, using the Bond Graph Approach. This alternative method of formulation of system dynamics, using Bond Graphs, offers a rich set of features that include, pictorial representation of the dynamics of translation and rotation for each link of the mechanism in the inertial frame, representation and handling of constraints at joints, depiction of *causality*, obtaining dynamic reaction forces and moments at various locations in the mechanism, algorithmic derivation of system equations in the first order *state-space* or *cause and effect* form, coding for simulation directly from the Bond Graph without deriving system equations, and so on. The links of mechanisms are modeled as rigid bodies and are suitably constrained to have certain desirable motion, or freedom, at joints. The multibond graph models have also been simulated using MATLAB code algorithmically and directly produced from them.

Keywords: Bond Graph, Modeling, Simulation, Mechanisms.

1 Modeling

Dynamics of two commonly used mechanisms, (1) the Crankshaft – Connecting rod – Piston – Cylinder system, and (2) the Universal Joint system, are modeled and simulated using the Bond Graph Approach. This alternative method of formulation of system dynamics, using Bond Graphs, offers a rich set of features [1, 2]. These include, pictorial representation of the dynamics of translation and rotation for each link of the mechanism in the inertial frame, depiction of *cause* and *effect* relationship, representation and handling of constraints at joints, obtaining the dynamic reaction forces and moments at various locations in the mechanism, derivation of system equations in the first order *state-space* or *cause and effect* form, coding for simulation directly from the Bond Graph without deriving system equations. Usually the links of mechanisms are modeled as rigid bodies. The links are further constrained to have certain desirable motion, or freedom, at joints. These constraints usually result in a revolute or prismatic configuration as intended by design.

In this work, we develop and apply a multibond graph model representing both translation and rotation of a rigid body for each link. The links are then coupled at joints based on the nature of constraint [3-5]. Both translational and rotational couplings for joints are developed and integrated with the dynamics of the connecting links. A problem of differential causality at link joints arises while modeling. This is rectified using additional stiffness and damping elements. It makes the model more realistic, bringing in effects of compliance and dissipation at joints, within definable tolerance limits. Multibond Graph models for the Crankshaft – Connecting rod – Piston – Cylinder system, and, the Universal Joint system [6], are developed using the Bond Graph Approach. Reference frames are fixed on each rigid link of the mechanisms using the Denavit-Hartenberg convention [7]. The translational effect is concentrated at the center of mass for each rigid link. Rotational effect is considered in the inertial frame itself, by considering the inertia tensor for each link about its respective center of mass, and expressed in the inertial frame. The multibond graph is then causaled and coding in MATLAB, for simulation, is carried out directly from the Bond Graph. A sketch of the crankshaft mechanism is shown in Fig.1, and its multibond graph model is shown in Fig.2. A sketch of the Universal joint system is shown in Fig.3, and its multibond graph model is shown in Fig.4. Results obtained from simulation of the dynamics of these mechanisms are then presented.

1.1 Crankshaft - Connecting Rod - Piston-Cylinder Mechanism

Fig. 1 shows the sketch of the “Crankshaft – Connecting rod – Piston – Cylinder system.”

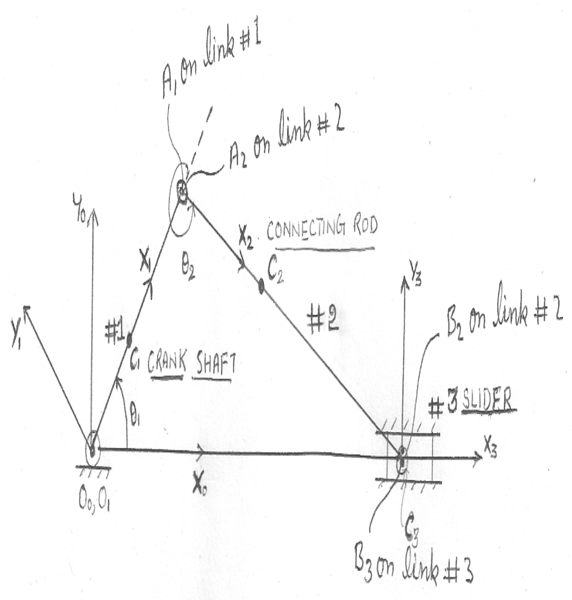

Fig. 1: Crankshaft-Connecting Rod-Piston-Cylinder Mechanism.

The individual components are considered as rigid links, connected at joints. The first moving link is the crank, the second link is the connecting rod and the third link is the piston. A frame is fixed on each link. Thus frame *1* is fixed on link *1*, frame *2* on link *2*, and frame *3* on link *3*. A fixed inertial frame *0*, whose origin coincides with frame *1*, is chosen. However, it will neither rotate nor translate. C_1, C_2 and C_3 are centres of mass of respective links. The frames are fixed on respective links using the Denavit-Hartenberg convention [4].

Dynamics of the system of Fig. 1 is modeled in the multibond graph shown in Fig. 2. The model depicts rotation as well as translation for each link in the system. The left side of the bond graph shows the rotational part and right part shows the translational part. We restrict any motion between the origin of inertial frame O and point on the link *1* that is O_1 by applying source of flow S_f as zero. Similarly we restrict any relative motion at point A, distinguished by A_1 on link *1* and A_2 on link *2*, by applying source of flow S_f as zero. The piston which is link 3, is constrained to translate only along the X_0 direction. Translation along Y_0 and Z_0 direction is constrained by applying source of flow S_f as zero for these components. Differential causality is eliminated by making the K(1,1) element of the stiffness matrix [K] between link *2* and link *3* as zero.

Additional stiffness and damping elements used for eliminating differential causality make the model more realistic, bringing in effects of compliance and dissipation at joints, within definable tolerance limits. These viscoelastic elements are represented in the bond graph by using **C** and **R** elements.

We have a source of effort S_e at link *3*, which is the pressure force acting on the piston, although this force is also acting only in *X* direction.

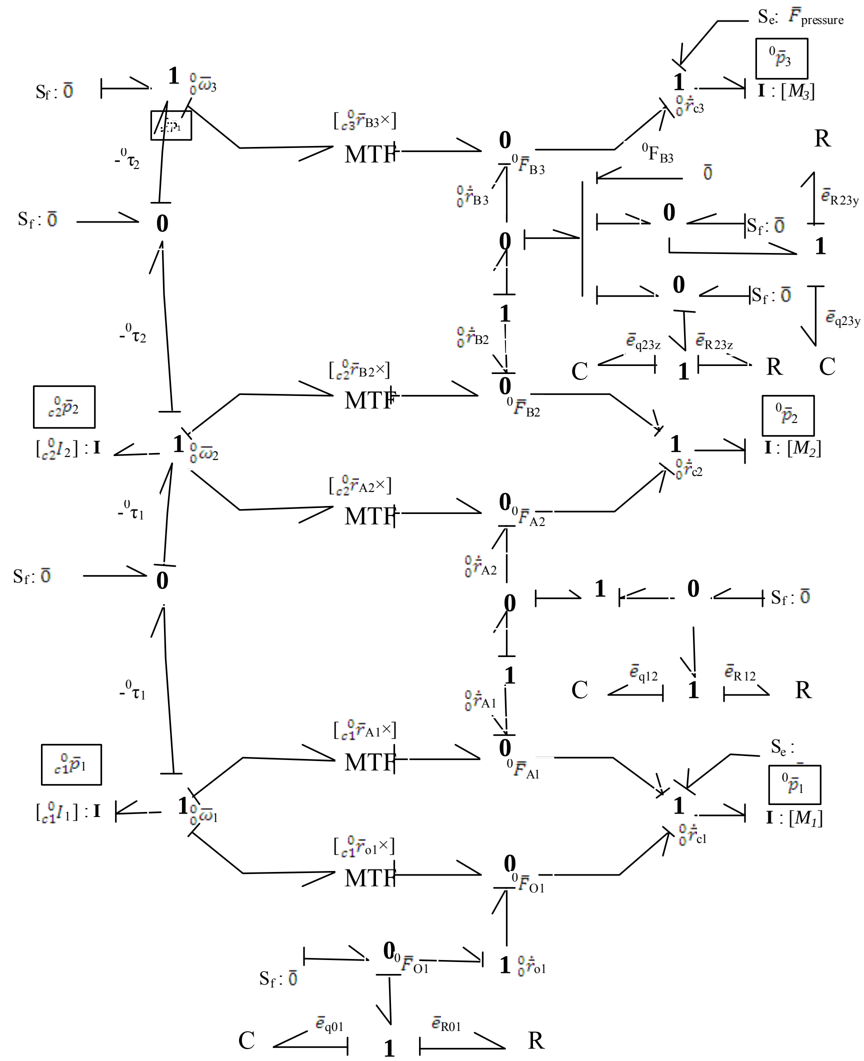

Fig. 2: Multibond graph model for the Crankshaft – Connecting rod – Piston – Cylinder system of Fig. 1.

1.2 Universal Joint Mechanism

The Fig. 3 shows the sketch of the " Universal Joint" mechanism.

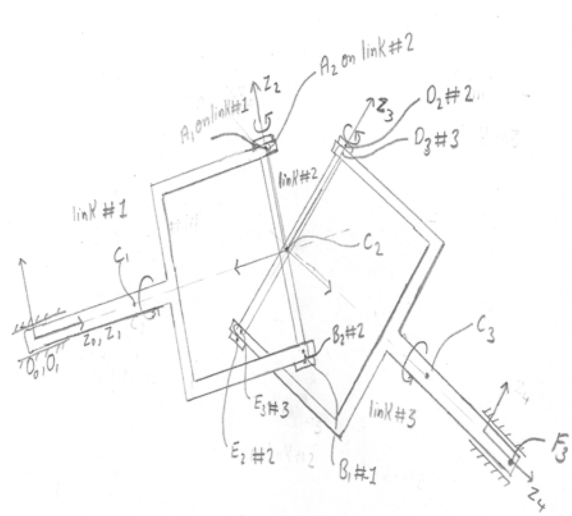

Fig. 3: Universal Joint Mechanism.

It has three rigid links, two are yokes which are attached to rotating shafts and the middle one is the cross con-

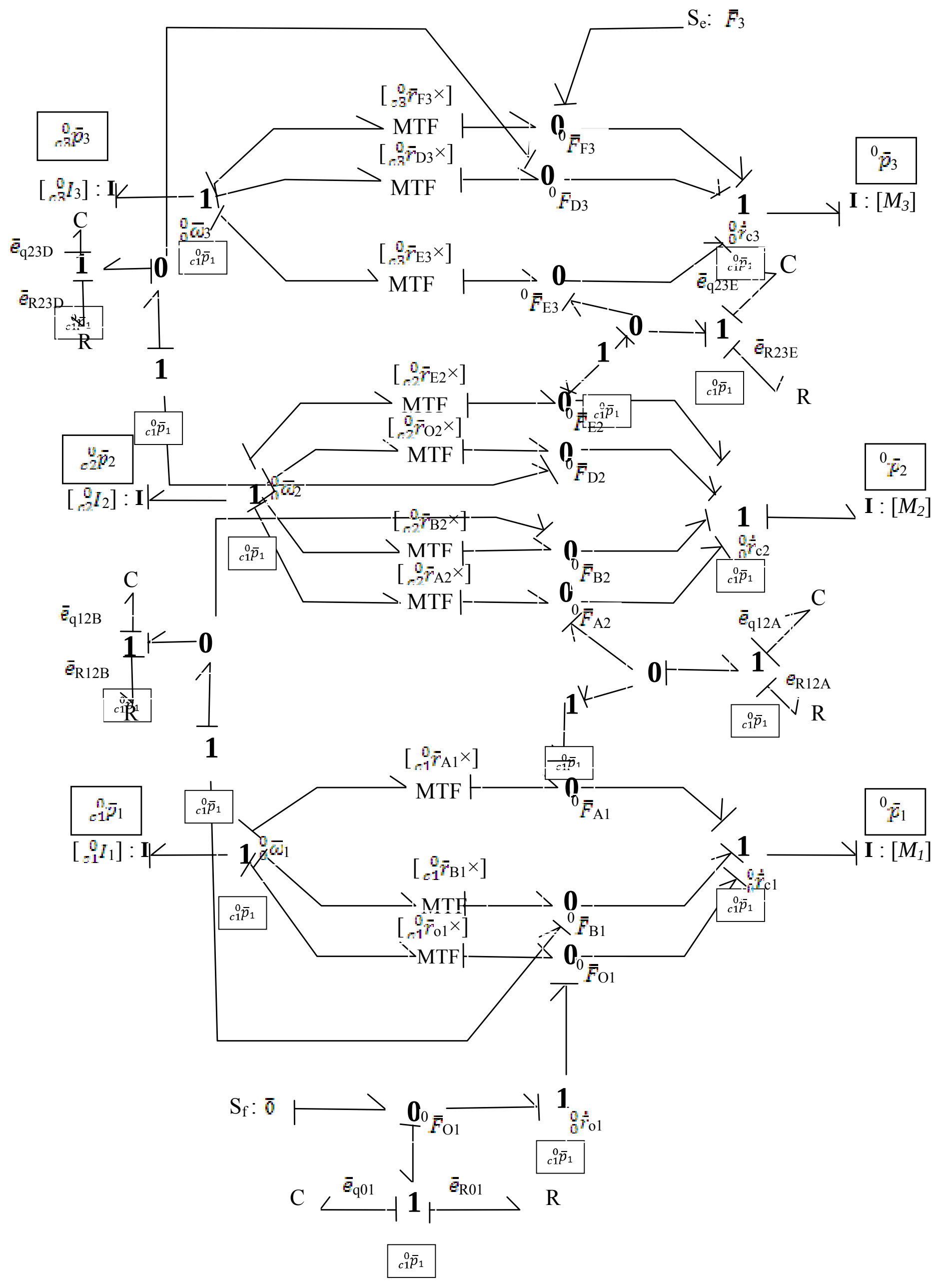

Fig. 4: Multibond graph for the Universal Joint system of Fig. 3.

necting the two yokes. The inertial frame is numbered 0, and it is fixed. Frame 1 is on link 1, frame 2 on the cross which is link 2, and frame 3 on the right yoke which is link 3. Origin of the inertial frame coincides with that of frame *1* of link *1*. The links *1* and *2* are connected with each other at two coincident end points points *A* - A_1 on link *1* and A_2 on link *2*, and *B* - B_1 on link *1* and B_2 on link *2*. Similarly links *2* and *3* are connected at two points *D* - D_2 on link *2* and D_3 on link *3*, and *E* - E_2 on link *2* and E_3 on link *3*.

Link *1* rotates about *Z* axis with respect to the inertial frame. The frame *2* is located at the centre of mass of the link *2*. Link *2* rotates with respect to the link *1* in direction Z_2 as shown in Fig. 3. Frame *3* also coincides with frame *2* but it is located on the link *2*. The frame *3* on link *3* rotates with respect to the link *2*, about Z_3, as shown in Fig. 3. The bond graph for this system is shown in Fig. 4.

The issue of differential causality arises for this mechanism also. It is eliminated using additional stiffness and damping elements. As discussed earlier, this makes the model more realistic, bringing in effects of compliance and dissipation at joints, within definable tolerance limits. The relative motion between the links at joints, along certain directions, is restrained by applying the source of flow S_f as zero. The constraint relaxation is tuned by changing the values of stiffness and damping at corresponding joints. Here we restrict the motion of the link *3* in two directions *Y* and *Z*, and allow motion in *X* direction by resolving the source of flow in three parts and by putting S_f as zero in Y and Z directions only.

For the simulation, an excitation torque is applied to link *1* about the Z direction.

2 Simulation

The results of computer simulation for the crankshaft mechanism of Fig. 1 are discussed first. The initial position of the crankshaft is at $\theta_1 = 60^\circ$ with the X_0 axis. It is then released under the effect of gravity. The force of gravity also acts on the connecting rod. No force due to gas pressure is considered for the simulation as it is not the main issue under focus for this paper. The upper row in Fig. 5 shows the displacement of the centre of mass C_1, as observed and expressed in Frame 0. It moves in a circular arc about the Z_0 axis. The first figure in the lower row of Fig. 5 shows the oscillation of the crankshaft about the Z_0 axis through change in orientation of the unit vectors of Frame *1*. The second figure in the second row shows the oscillation of the centre of mass C_1 with time. It is interesting to observe that the oscillations appear to reduce in amplitude with passage of time although no damping is considered. This could perhaps be ascribed to the nonlinearity imposed due to coupling with the connecting rod.

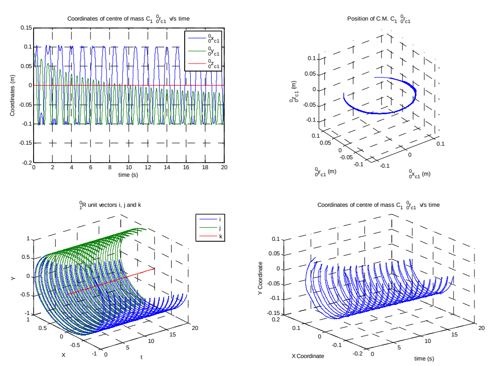

Fig. 5: Response motion of the crankshaft.

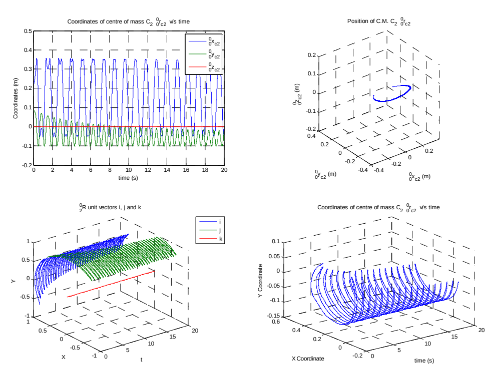

Fig. 6: Response motion of the connecting rod.

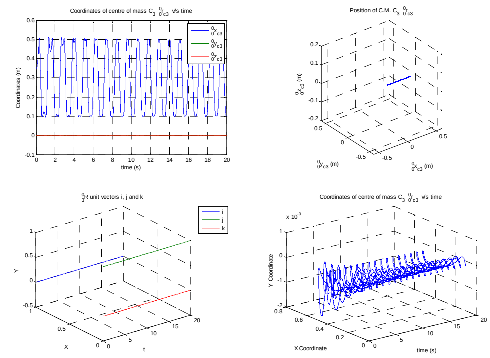

Fig. 7: Response motion of the piston.

Simulation results for the Universal joint system are presented in Fig. 8. A constant torque is applied to the driving shaft about its axis. The driven shaft makes an angle of 5° with the axis of the driving shaft. The First row shows the response of the driving shaft which is the first link. The component of angular momentum of the driving shaft about its axis increases linearly, which is as expected. The first two figures of the second row show the change in orientation of the cross, which is link 2. Angular motion about all three axes is clearly visible. Change in orientation of the unit vector ${}^0\hat{i}_3$ is small and corresponds to a small circle, due to the misalignment of 5°. The driven shaft follows the motion of the driver shaft as is clear from the third row in Fig. 8.

3 Conclusions

The Bond Graph approach is used to model dynamics of two commonly used mechanisms, (1) the Crankshaft – Connecting rod – Piston – Cylinder system, and (2) the Universal Joint system. Pictorial representation of the dynamics of translation and rotation for each link of the mechanism in the inertial frame, representation and handling of constraints at joints, depiction of *cause* and *effect* relationships, coding for simulation directly from the Bond Graph without deriving system equations, have been explained in this work. MATLAB based simulations have been presented and interpreted for both the systems.

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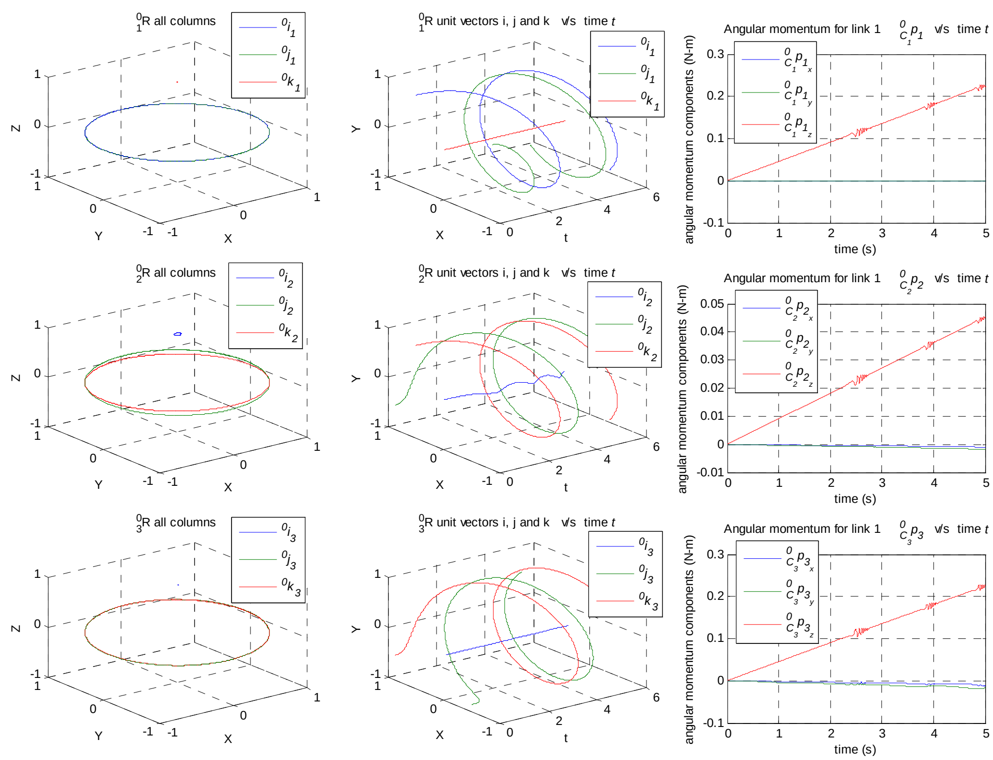

8: Simulation results for the Universal joint system.