Free Vibration Analysis of Bimodular Material Laminated Thick Plates Using an Efficient Individual Layer Theory

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Abstract

The effect of bimodularity on free vibration of all edges simply supported, two-layered, cross-ply thick plates are investigated by using Berti's constitutive material model. An effecttive layerwise laminate theory has been used to analyze the free vibration behavoir by anlytical approach. The free vibration fundamental frequencies for various bimodularity ratios, aspect ratios and side to thickness ratios are presented. The through thickness fiber direction strain , in-plane stresses and trnsverse shear stresses distribution for a typical case is shown.

Keywords: Bimodular, Bert's model, Effective layer wise theory.

1 Introduction

Bimodularity is the different behavior of material in tension and compression as shown in Fig. (1). Apart from certain fiber reinforced composites, bone and some biological tissues too exhibit bimodularity. The static analysis of bimodular plates is carried out by Cho et al [1,2]. The free vibration analysis of either plate or panel is carried out either by using first order shear deformation theory or using third order theory by a few researchers [3-6]. The forced response analysis of bimodular panel is carried out by present authors [7-10].

In this paper an efficient individual layer wise theory and Bert's constitutive model is used to study the effect of bimodularity, aspect and thickness ratios on the free vibration characteristics of bimodular laminated all edges simply supported cross-ply plates by analytical method.

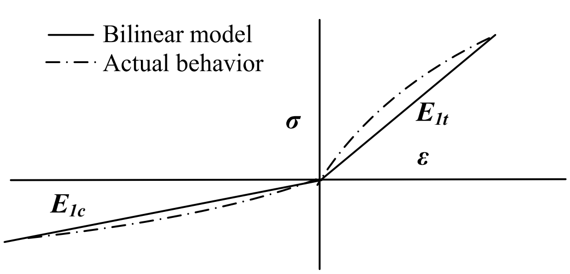


Fig. 1: Stress-strain behavior of bimodulus material.

2 Formulation

A cross-ply composite plate is considered with the co-ordinates *x*, *y* along the in-plane directions and *z* along the thickness direction with the dimensions *a*, *b*, *h* along *x, y* and *z* directions, respectively, as shown in Fig. (2). The in-plane and transverse displacements for k^{th} layer are assumed as:

$$u^k(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w}{\partial x} + \left(f_1 + g_1^k\right)\left(\frac{\partial w}{\partial x} + \theta_x(x,y,t)\right) + g_2^k\left(\frac{\partial w}{\partial y} + \theta_y(x,y,t)\right)$$

$$v^k(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w}{\partial y} + g_3^k\left(\frac{\partial w}{\partial x} + \theta_x(x,y,t)\right) + \left(f_2 + g_4^k\right)\left(\frac{\partial w}{\partial y} + \theta_y(x,y,t)\right)$$

$$w^k(x,y,z,t) = w_0(x,y,t) \quad (1)$$

where

$$f_1 = \frac{h}{\pi}\left(\sin\frac{\pi z}{h} - b_{44}\cos\frac{\pi z}{h}\right)$$

$$f_2 = \frac{h}{\pi}\left(\sin\frac{\pi z}{h} - b_{55}\cos\frac{\pi z}{h}\right)$$

$$g_1^k = a_1^k z + d_1^k$$

$$g_2^k = a_2^k z + d_2^k - \frac{h}{\pi}a_{44}\cos\frac{\pi z}{h}$$

$$g_3^k = a_3^k z + d_3^k - \frac{h}{\pi}a_{55}\cos\frac{\pi z}{h}$$

$$g_4^k = a_4^k z + d_4^k$$

Here u_0, v_0, w_0 are the displacements of mid-surface (z =0) and θ_x,θ_y are the rotations of normal to the mid-plane about the *y* and *x* axes, respectively. In this model, there are $8N_e$+4 $\left(4N_e : d_i^k, 4N_e : a_i^k, a_{44}, a_{55}, b_{44}, b_{55}; i = 1,2,3,4\right)$ constants which need to be determined where N_e is number of effective layers. If *n* is number of layers which have partly tensile properties and partly compressive properties and N is total number of layers, then $N_e=N+n$. To determine the unknown constants, the following conditions are satisfied

$$u^{k+1} \text{ at } z_{k+1} = u^k \text{ at } z_{k+1}$$

$$v^{k+1} \text{ at } z_{k+1} = v^k \text{ at } z_{k+1} \quad \text{for } k = 1 \text{ to } k = N_e - 1$$

$$u^{N/2} = v^{N/2} = 0 \text{ at } z_{\frac{N}{2}+1} = 0 \,(\text{assuming } N \text{ is even})$$

$$\tau_{yz} = \tau_{xz} = 0 \text{ at } z_1 = -\frac{h}{2} \text{ and } z_{N_e+1} = \frac{h}{2}$$

$$\tau_{yz}^{k+1}(z_{k+1}) = \tau_{yz}^{k}(z_{k+1}) \text{ for } k = 1 \text{ to } k = N_e - 1$$

$$\tau_{xz}^{k+1}(z_{k+1}) = \tau_{xz}^{k}(z_{k+1}) \text{ for } k = 1 \text{ to } k = N_e - 1$$

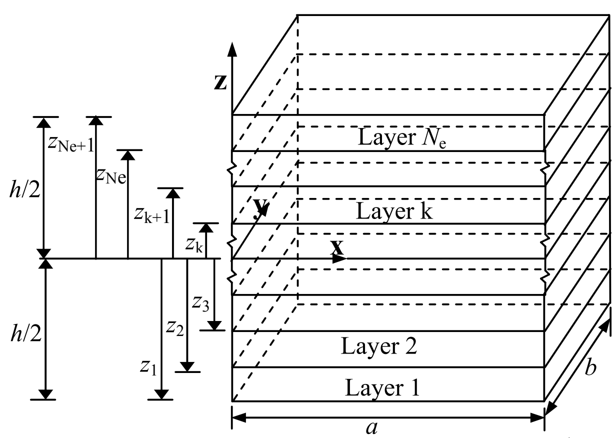


Fig. 2: Geometry of a rectangular laminated plate.

The boundary conditions considered are:

$v_0 = w_0 = \theta_y = 0$ at $x = 0, a$.

$u_0 = w_0 = \theta_x = 0$ at $y = 0, b$. (2)

The solution satisfying the boundary condition [Eq. (2)] is taken as:

$$u_0 = \sum_{j=1}^{P}\sum_{i=1}^{Q} u_{0ij} \,\text{Cos}\frac{i\pi x}{a}\,\text{Sin}\frac{j\pi y}{b}$$

$$v_0 = \sum_{j=1}^{P}\sum_{i=1}^{Q} v_{0ij} \,\text{Sin}\frac{i\pi x}{a}\,\text{Cos}\frac{j\pi y}{b}$$

$$w_0 = \sum_{j=1}^{P}\sum_{i=1}^{Q} w_{0ij} \,\sin\frac{i\pi x}{a}\,\sin\frac{j\pi y}{b}$$

$$\theta_x = \sum_{j=1}^{P}\sum_{i=1}^{Q} \theta_{xij} \,\sin\frac{i\pi x}{a}\,\cos\frac{j\pi y}{b}$$

$$\theta_y = \sum_{j=1}^{P}\sum_{i=1}^{Q} \theta_{yij} \,\cos\frac{i\pi x}{a}\,\sin\frac{j\pi y}{b} \quad (3)$$

For fundamental mode of vibration $P = Q = 1$ is sufficient to get the accurate results.

Based on fiber direction strain governed model, the constitutive relation of k^{th} layer of a bimodulus laminated cross-ply plate can be written as:

$$\left\{\boldsymbol{\sigma}^k\right\} = \begin{Bmatrix} \sigma_{xx}^k \\ \sigma_{yy}^k \\ \tau_{xy}^k \\ \tau_{yz}^k \\ \tau_{xz}^k \end{Bmatrix} = \begin{Bmatrix} \left\{\boldsymbol{\sigma}_{\mathbf{p}}^k\right\} \\ \left\{\boldsymbol{\sigma}_{\mathbf{s}}^k\right\} \end{Bmatrix} =$$

$$\begin{bmatrix} \overline{Q}_{11l}^k & \overline{Q}_{12l}^k & 0 & 0 & 0 \\ \overline{Q}_{12l}^k & \overline{Q}_{22l}^k & 0 & 0 & 0 \\ 0 & 0 & \overline{Q}_{66l}^k & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44l}^k & 0 \\ 0 & 0 & 0 & 0 & \overline{Q}_{55l}^k \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^k \\ \varepsilon_{yy}^k \\ \gamma_{xy}^k \\ \gamma_{yz}^k \\ \gamma_{xz}^k \end{Bmatrix} \quad (4)$$

where $\overline{Q}_{ijl}^k$ are transformed stiffness coefficient and k is layer number, $l = 1$ denotes the properties associated with fiber direction tension and $l = 2$ denotes the properties associated with fiber direction compression,

$$\left\{\boldsymbol{\sigma}_{\mathbf{p}}^k\right\} = \begin{Bmatrix} \sigma_{xx}^k \\ \sigma_{yy}^k \\ \tau_{xy}^k \end{Bmatrix} \text{ and } \left\{\boldsymbol{\sigma}_{\mathbf{s}}^k\right\} = \begin{Bmatrix} \tau_{yz}^k \\ \tau_{xz}^k \end{Bmatrix} \quad (5)$$

The strain vector can be written as:

$$\left\{\boldsymbol{\varepsilon}^k\right\} = \begin{Bmatrix} \varepsilon_{xx}^k \\ \varepsilon_{yy}^k \\ \gamma_{xy}^k \\ \gamma_{yz}^k \\ \gamma_{xz}^k \end{Bmatrix} = \begin{Bmatrix} \left\{\boldsymbol{\varepsilon}_{\mathbf{p}}^k\right\} \\ \left\{\boldsymbol{\varepsilon}_{\mathbf{s}}^k\right\} \end{Bmatrix} \quad (6)$$

$$\text{where } \left\{\boldsymbol{\varepsilon}_{\mathbf{p}}^k\right\} = \begin{Bmatrix} \varepsilon_{xx}^k \\ \varepsilon_{yy}^k \\ \gamma_{xy}^k \end{Bmatrix} \text{ and } \left\{\boldsymbol{\varepsilon}_{\mathbf{s}}^k\right\} = \begin{Bmatrix} \gamma_{yz}^k \\ \gamma_{xz}^k \end{Bmatrix} \quad (7)$$

The strain energy of the plate is given by:

$$U\{\delta\} = \frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}}\left\{\boldsymbol{\sigma}^k\right\}^{\mathrm{T}}\left\{\boldsymbol{\varepsilon}^k\right\}\mathrm{d}z\right]\mathrm{d}x\,\mathrm{d}y \quad (8)$$

Using Eqs. (4), (5) and (6), Eq. (8) can be rewritten as:

$$U\{\delta\} = \frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}}\left\{\boldsymbol{\sigma}_{\mathbf{p}}^k\right\}^{\mathrm{T}}\left\{\boldsymbol{\varepsilon}_{\mathbf{p}}^k\right\}\mathrm{d}z\right]\mathrm{d}x\mathrm{d}y +$$

$$\frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}}\left\{\boldsymbol{\sigma}_{\mathbf{s}}^k\right\}^{\mathrm{T}}\left\{\boldsymbol{\varepsilon}_{\mathbf{s}}^k\right\}\mathrm{d}z\right]\mathrm{d}x\mathrm{d}y$$

$$= \frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}}\left\{\boldsymbol{\varepsilon}_{\mathbf{p}}^k\right\}^{\mathrm{T}}\left[\mathbf{Q}_{ijl}'^{k}\right]\left\{\boldsymbol{\varepsilon}_{\mathrm{p}}^k\right\}\mathrm{d}z\right]\mathrm{d}x\mathrm{d}y +$$

$$\frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}}\left\{\boldsymbol{\varepsilon}_{\mathbf{s}}^k\right\}^{\mathrm{T}}\left[\mathbf{Q}_{ijl}''^{k}\right]\left\{\boldsymbol{\varepsilon}_{\mathbf{s}}^k\right\}\mathrm{d}z\right]\mathrm{d}x\mathrm{d}y \quad (9)$$

where $\left[\mathbf{Q}'^{k}_{ijl}\right]=\begin{bmatrix}\overline{Q}^{k}_{11l} & \overline{Q}^{k}_{12l} & 0\\ \overline{Q}^{k}_{12l} & \overline{Q}^{k}_{22l} & 0\\ 0 & 0 & \overline{Q}^{k}_{66l}\end{bmatrix}$ and

$$\left[\mathbf{Q}''^{k}_{ijl}\right]=\begin{bmatrix}\overline{Q}^{k}_{44l} & 0\\ 0 & \overline{Q}^{k}_{55l}\end{bmatrix}$$

Using Eqs. (1) and (3), $\{\boldsymbol{\varepsilon}^{k}_{\mathbf{p}}\}$ and $\{\boldsymbol{\varepsilon}^{k}_{\mathbf{s}}\}$ can be written as:

$$\{\boldsymbol{\varepsilon}^{k}_{\mathbf{p}}\}=[\mathbf{Z_p}][\mathbf{T_p}]\{\boldsymbol{\delta}\},\ \{\boldsymbol{\varepsilon}^{k}_{\mathbf{s}}\}=[\mathbf{Z_s}][\mathbf{T_s}]\{\boldsymbol{\delta}\} \qquad (10)$$

where

$$[\mathbf{Z_p}]_{3X11}=\left[[\mathbf{Z_1}]_{3X6}\quad [\mathbf{Z_2}]_{3X5}\right] \text{ and } [\mathbf{Tp}]_{11X5}=\begin{bmatrix}[\mathbf{Z_3}]_{4X2} & [\mathbf{Z_4}]_{4X3}\\ [\mathbf{Z_5}]_{7X2} & [\mathbf{Z_6}]_{7X3}\end{bmatrix}$$

$$[\mathbf{Z_1}]=\begin{bmatrix}1 & 0 & 0 & 0 & -z & 0\\ 0 & 0 & 0 & 1 & 0 & -z\\ 0 & 1 & 1 & 0 & 0 & 0\end{bmatrix}$$

$$[\mathbf{Z_2}]=\begin{bmatrix}0 & f_1+g^k_1 & 0 & g^k_2 & 0\\ 0 & 0 & g^k_3 & 0 & f_2+g^k_4\\ -2z & g^k_3 & f_1+g^k_1 & f_2+g^k_4 & g^k_2\end{bmatrix}$$

$$[\mathbf{Z_3}]=\begin{bmatrix}-\frac{\pi\, ss}{a} & 0\\ \frac{\pi\, sc}{b} & 0\\ 0 & \frac{\pi\, cc}{a}\\ 0 & -\frac{\pi\, ss}{b}\end{bmatrix}$$

$[\mathbf{Z_4}]$ and $[\mathbf{Z_5}]$ are null matrices.

$$[\mathbf{Z_6}]=\begin{bmatrix}-\left(\frac{\pi}{a}\right)^2 ss & 0 & 0\\ -\left(\frac{\pi}{b}\right)^2 ss & 0 & 0\\ \left(\frac{\pi^2}{ab}\right) cc & 0 & 0\\ -\left(\frac{\pi}{a}\right)^2 ss & -\left(\frac{\pi}{a}\right) ss & 0\\ \left(\frac{\pi^2}{ab}\right) cc & \left(\frac{\pi}{b}\right) cc & 0\\ \left(\frac{\pi^2}{ab}\right) cc & 0 & \left(\frac{\pi}{a}\right) cc\\ -\left(\frac{\pi}{b}\right)^2 ss & 0 & -\left(\frac{\pi}{b}\right) ss\end{bmatrix}$$

$$[\mathbf{Z_s}]=\begin{bmatrix}\frac{\partial g^k_3}{\partial z} & \frac{\partial\left(f_2+g^k_4\right)}{\partial z}\\ \frac{\partial\left(f_1+g^k_1\right)}{\partial z} & \frac{\partial g^k_2}{\partial z}\end{bmatrix}$$

$$[\mathbf{T_s}]=\begin{bmatrix}0 & 0 & \frac{\pi\, sc}{b} & sc & 0\\ 0 & 0 & \frac{\pi\, cs}{a} & 0 & cs\end{bmatrix}$$

$$sc=\sin\frac{\pi x}{a}\cos\frac{\pi y}{b}$$

$$cs=\sin\frac{\pi y}{b}\cos\frac{\pi x}{a}$$

$$ss=\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$

$$cc=\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}$$

and $\{\boldsymbol{\delta}\}^{\mathrm{T}}=\{u_{011}\quad v_{011}\quad w_{011}\quad \theta_{x11}\quad \theta_{y11}\}$

Using Eq. (10), Eq. (9) can be rewritten as:

$$U\{\delta\}=$$

$$\frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}}\{\boldsymbol{\delta}\}^{\mathrm{T}}[\mathbf{T_p}]^{\mathrm{T}}[\mathbf{Z_p}]^{\mathrm{T}}\left[\mathbf{Q}'^{k}_{ijl}\right][\mathbf{Z_p}][\mathbf{T_p}]\{\boldsymbol{\delta}\}\mathrm{d}z\right]\mathrm{d}x\mathrm{d}y+$$

$$\frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}}\{\boldsymbol{\delta}\}^{\mathrm{T}}[\mathbf{T_s}]^{\mathrm{T}}[\mathbf{Z_s}]^{\mathrm{T}}\left[\mathbf{Q}''^{k}_{ijl}\right][\mathbf{Z_s}][\mathbf{T_s}]\{\boldsymbol{\delta}\}\mathrm{d}z\right]\mathrm{d}x\mathrm{d}y \qquad (11)$$

The kinetic energy of plate is:

$$T\{\delta\}=$$

$$\frac{1}{2}\int_0^b\int_0^a\left[\sum_{k=1}^{N_e}\int_{h_k}^{h_{k+1}}\rho^k\{\dot{u}^k_0\quad \dot{v}^k_0\quad \dot{w}^k_0\}\{\dot{u}^k_0\quad \dot{v}^k_0\quad \dot{w}^k_0\}^{\mathrm{T}}\mathrm{d}z\right]\mathrm{d}x\,\mathrm{d}y \qquad (12)$$

Using Eqs. (1) and (3), $\{\dot{u}^k_0\quad \dot{v}^k_0\quad \dot{w}^k_0\}^{\mathrm{T}}$ can be written as:

$$\{\dot{u}^k_0\quad \dot{v}^k_0\quad \dot{w}^k_0\}^{\mathrm{T}}=[\mathbf{Z_m}][\mathbf{T_m}]\{\dot{\boldsymbol{\delta}}\} \qquad (13)$$

where $\{\dot{\boldsymbol{\delta}}\}^{\mathrm{T}}=\{\dot{u}_{011}\quad \dot{v}_{011}\quad \dot{w}_{011}\quad \dot{\theta}_{x11}\quad \dot{\theta}_{y11}\}$

$$[\mathbf{Z_m}]=\begin{bmatrix}1 & 0 & 0 & -z & 0 & f_1+g^k_1 & g^k_2\\ 0 & 1 & 0 & 0 & -z & g^k_3 & f_2+g^k_4\\ 0 & 0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$[\mathbf{T_m}]=\begin{bmatrix}cs & 0 & 0 & 0 & 0\\ 0 & sc & 0 & 0 & 0\\ 0 & 0 & ss & 0 & 0\\ 0 & 0 & \frac{\pi\, cs}{a} & 0 & 0\\ 0 & 0 & \frac{\pi\, sc}{b} & 0 & 0\\ 0 & 0 & \frac{\pi\, cs}{a} & cs & 0\\ 0 & 0 & \frac{\pi\, sc}{b} & 0 & sc\end{bmatrix}$$

Using Eq. (13), Eq. (12) can be rewritten as:

$$T\{\delta\} = \frac{1}{2}\int_0^b\int_0^a \left[\sum_{k=1}^{N_e}\int_{z_k}^{z_{k+1}} \rho^k \{\dot{\boldsymbol{\delta}}\}^{\mathrm{T}} [\mathbf{T_m}]^{\mathrm{T}} [\mathbf{Z_m}]^{\mathrm{T}} [\mathbf{Z_m}][\mathbf{T_m}] \{\dot{\boldsymbol{\delta}}\} \mathrm{d}z\right] \mathrm{d}x\, \mathrm{d}y \quad (14)$$

Using above potential and kinetic energy expressions in Lagrange's equation of motion, the governing equation is obtained as:

$$[\mathbf{M}]\{\ddot{\boldsymbol{\delta}}\} + [\mathbf{K}]\{\boldsymbol{\delta}\} = \{0\} \quad (15)$$

Assuming the solution $\{\boldsymbol{\delta}\} = \{\overline{\boldsymbol{\delta}}\} e^{I\omega t}$ $\left(I = \sqrt{-1}\right)$ for free vibration analysis, the Equation (4) can be rewritten as:

$$-\omega^2[\mathbf{M}]\{\overline{\boldsymbol{\delta}}\} + [\mathbf{K}]\{\overline{\boldsymbol{\delta}}\} = \{0\} \quad (16)$$

The free vibration frequencies are extracted using iterative eigenvalue approach from Eq. (16).

3 Results and Discussions

The material properties considered are:

In tension: $E_{1t}/E_{2t} = 25$, $E_{2t} = E_{3t}$, $E_{3t} = E_{2t}$, $G_{12t}/E_{2t} = G_{13t}/E_{2t} = 0.5$, $G_{23t}/E_{2t} = 0.2$, $\nu_{12t} = \nu_{23t} = \nu_{13t} = 0.25$.

In compression: $E_{1c}/E_{2c} = 25$, $E_{2c} = E_{3c} = 1$ GPa, $G_{12c}/E_{2c} = G_{13c}/E_{2c} = 0.5$, $G_{23c}/E_{2c} = 0.2$,

$\nu_{12c} = \nu_{23c} = \nu_{13c} = 0.25$. E_{2t}/E_{2c} is varied from 0.2 to 2.

The through thickness non-dimensional transverse shear stress (S_{yz}, S_{xz}) distribution for a two-layered cross-ply bimodular plate (b/h=10, a/b=1) for sinusoidally distributed transverse load are compared with the Ref. [2] and presented in Fig. (3), which shows very good agreement with the present results.

The fiber direction strain [$\varepsilon_{11}(a/2, b/2, z)$] distribution of two-layered cross-ply plate (a/b=0.5, b/h=5) for positive and negative half cycle is shown in Fig. (4) for E_{2t}/E_{2c}=0.2 and E_{2t}/E_{2c} =2.0. The strain distribution for positive and negative half cycle is completely different and also, the negative half cycle strain distribution is not achievable by just multiplying the positive half cycle by -1, which is the indication of different stiffness matrix for positive and negative half cycle and hence different frequencies for positive (ω_1) and negative (ω_2) half cycles.

The through thickness in-plane normal [$S_{xx}(a/2, b/2, z)$, $S_{yy}(a/2, b/2, z)$] stresses distribution for positive and negative half cycles are shown in Figs. (5) and (6) for different bimodularity ratios. The distribution patterns are non-linear and due to bimodularity the stress are discontinuous in a lamina (where the lamina is partly under tension and partly under compression along fiber direction) unlike unimodular case where the in-plane stresses are continuous in a lamina.

The in-plane shear stress [S_{xy} (0, 0, z)] distribution for positive and negative half cycles is shown in Fig. (7).The distribution pattern is nonlinear and discontinuity in a lamina is observed.

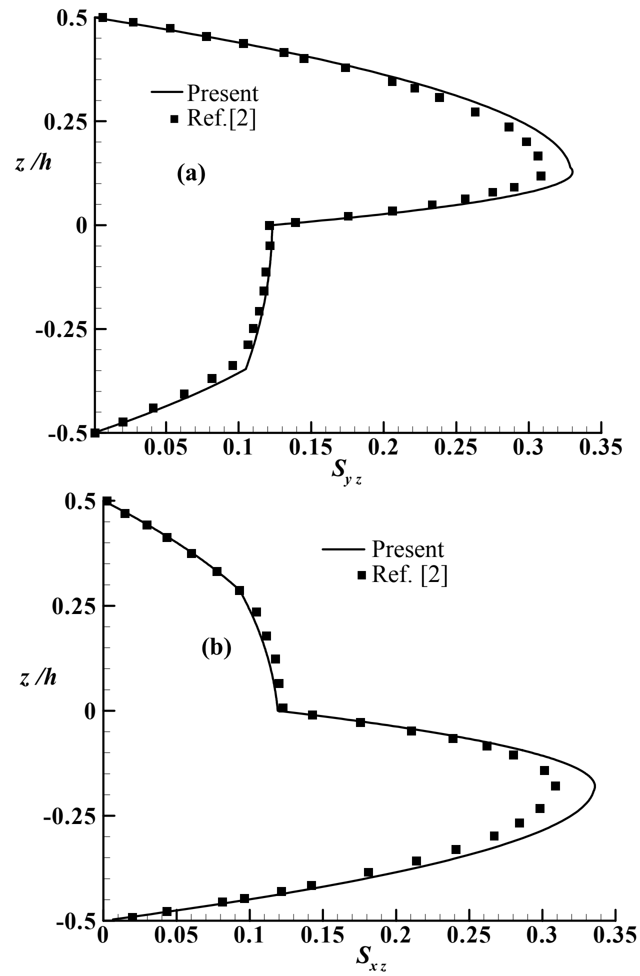


Fig. 3: Comparison of through thickness transverse shear stresses distribution for two-layered cross-ply bimodular laminate: (a) S_{yz} (a/2, 0, z), (b) S_{xz} (0, b/2, z).

The transverse shear stress [S_{yz} (a/2, 0, z), S_{xz} (0, b/2, z)] distribution is shown in Fig. (8) for positive and negative half cycles for different bimodularity ratios. The stress distribution is nonlinear and stress vanishes at the top and bottom of the laminate. Here also the negative half cycle stress can not be obtained by just multiplying the positive half cycle stress by -1. As the bimodularity increases the stresses increase.

The fundamental non-dimensional positive and negative half cycle frequencies [Ω_1, Ω_2 = (ω_1, ω_2) b^2 $(\rho/E_{2c}/h^2)^{1/2}$] versus bimodularity ratios (E_{2t}/E_{2c}) is plotted in Fig. (9) for different aspect- and thickness ratios of bimodular plate. The difference of positive and negative half cycle frequencies is greater for a/b=0.5 compared to a/b=2. The positive and negative half cycle frequencies are same for square plate irrespective of bimodularity ratios. These Figs also indicate that the plate is thinner the difference is less. As the bimodularity increases the frequency parameters increases

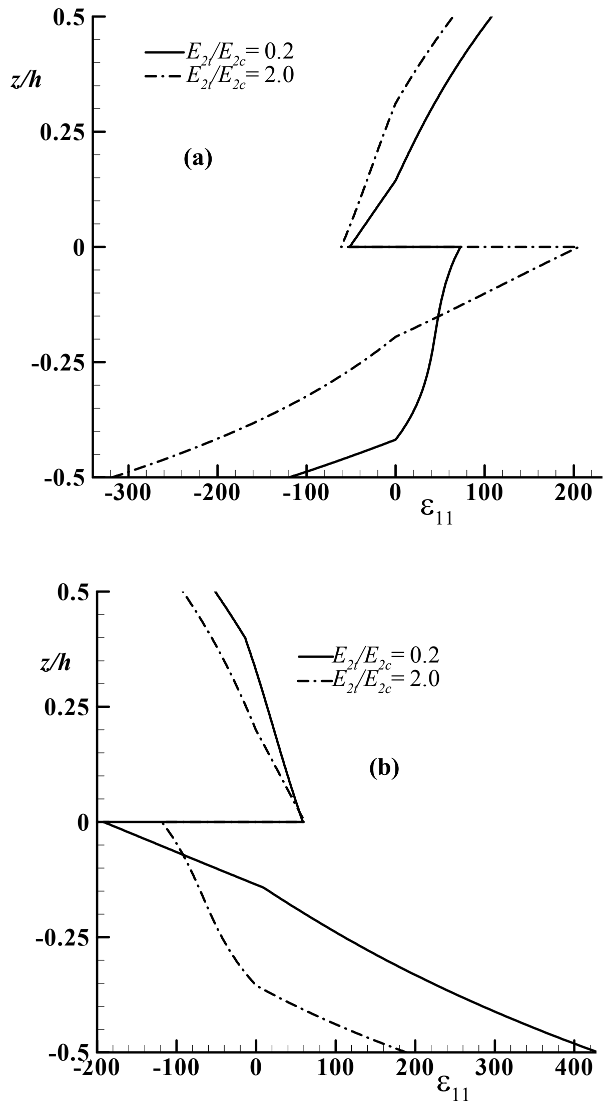
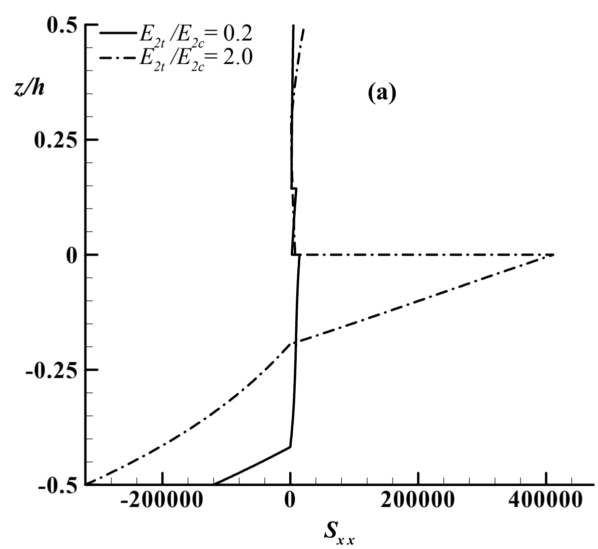


Fig. 4: Fiber direction strain (ε_{11}) distribution of bimodular plate (*a/b*=0.5, *b/h*=5, 0º/90º): (a) Positive half cycle, (b) Negative half cycle.



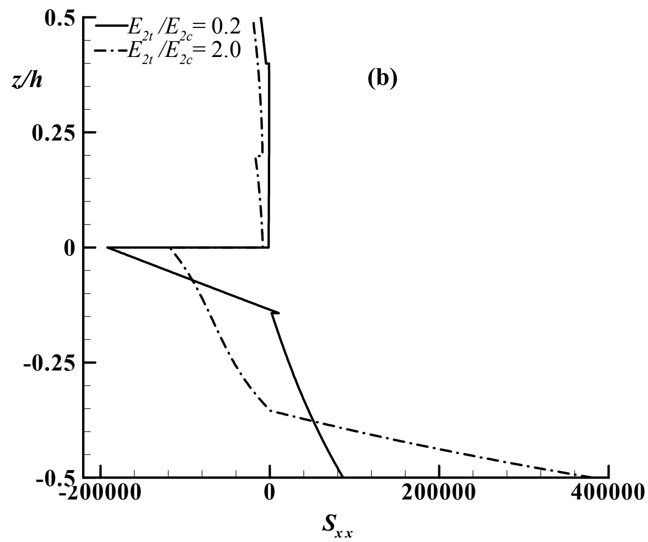
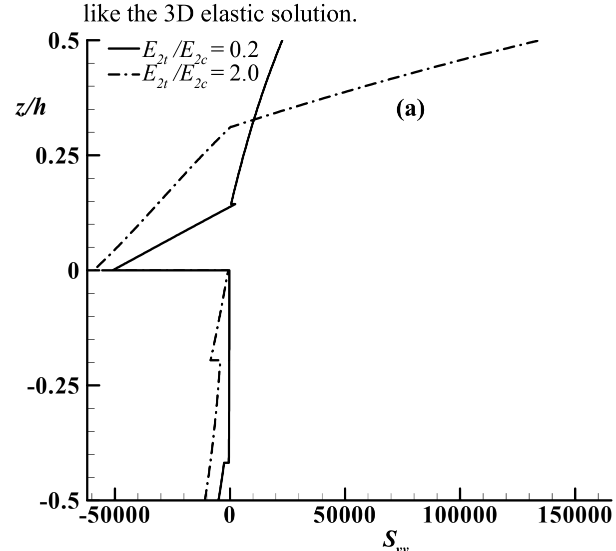


Fig. 5: Through thickness normal stress (S_{xx}) distribution of bimodular laminate (*a/b*=0.5, *b/h*=5, 0º/90º): (a) Positive half cycle, (b) Negative half cycle.

Conclusions

From the above discussions the following conclusions can be drawn.

1) The positive and negative half cycle frequencies are different for rectangular plate for $E_{2t}/E_{2c} \neq 1$ and are same for $E_{2t}/E_{2c}=1$. For square plate positive and negative half cycle frequencies are same.
2) The through thickness stresses distribution for negative cycle can not be obtained by multiplying the positive cycle distribution by -1 and vice- versa.
3) There will be discontinuity of in-plane stress in a lamina if the lamina has partly tensile strain and partly compressive strain along the fiber direction.
4) The transverse shear stresses are continuous and vanish at the top and bottom of the laminate like the 3D elastic solution.



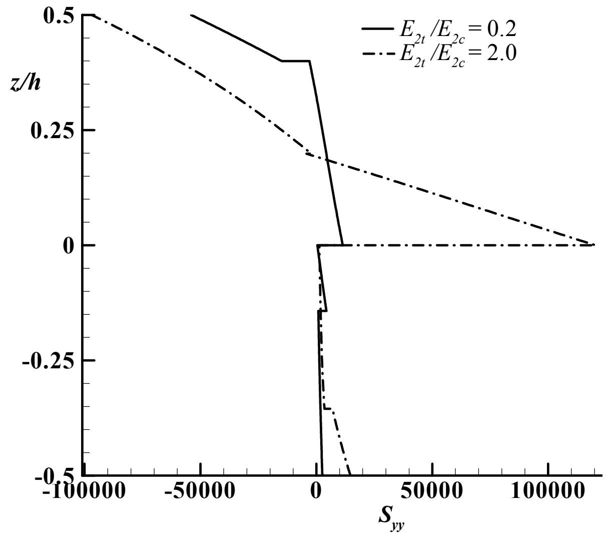


Fig. 6: Through thickness normal stress (S_{yy})distribution for bimodular laminate (*a/b*=0.5, *b/h*=5, 0º/90º): (a) Positive half cycle, (b) Negative half cycle.

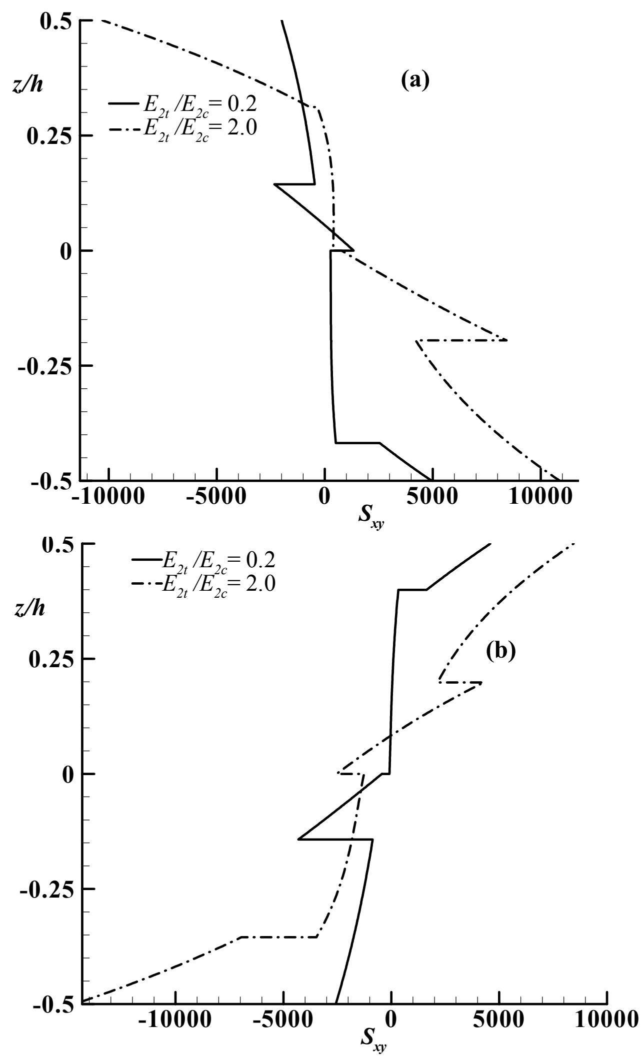


Fig. 7: Through thickness in-plane shear stress (S_{xy}) distribution of bimodular laminate (*a/b*=0.5, *b/h*=5, 0º/90º): (a) Positive half cycle, (b) Negative half cycle.

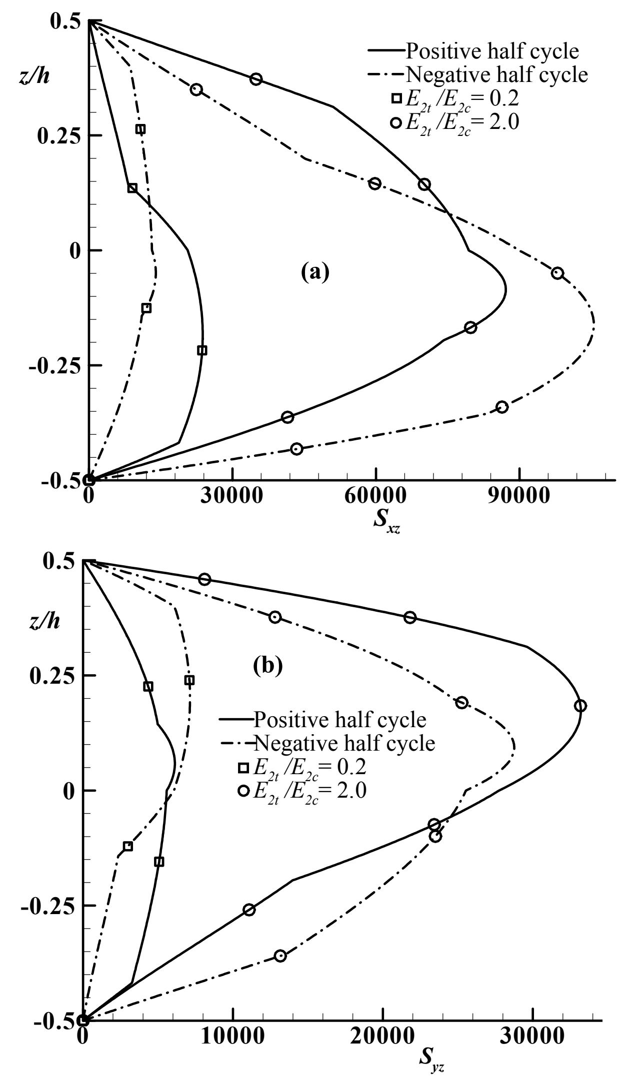
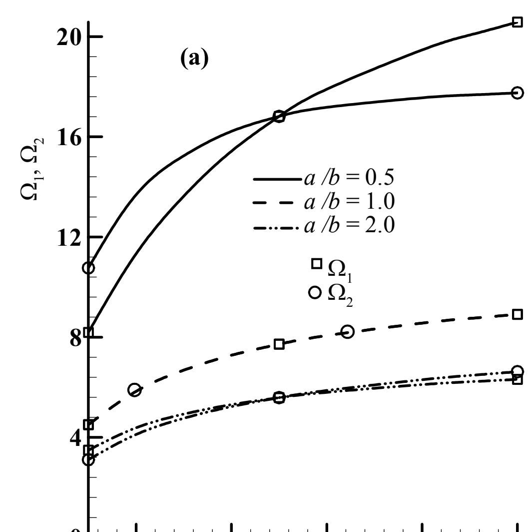


Fig. 8: Through thickness transverse shear stress distribution for bimodular laminate (*a/b*=0.5, *b/h*=5, 0º/90º): (a) S_{xz}, (b) S_{yz}.



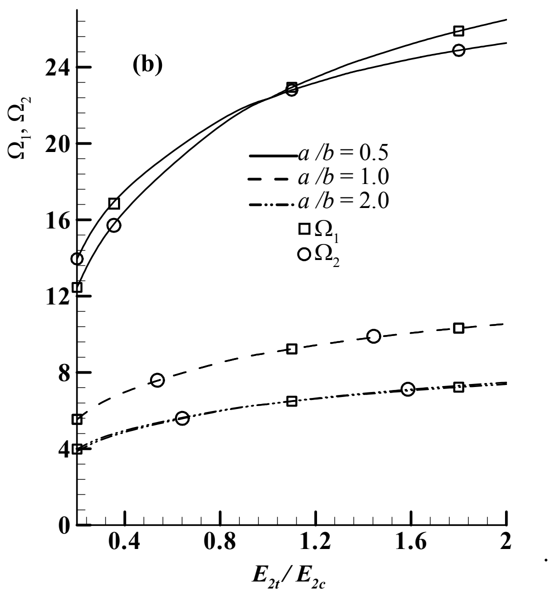


Fig. 9: Variation of non-dimensionalized positive and negative half cycle frequencies for two layered cross-ply (0º/90º) plates: (a) *b/h*=5, (b) *b/h*=10.

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References

[1] K. N. Cho et al, "Bending Analysis of Thick Bimodular Laminates by Higher-order Individual-layer Theory," *Composite Structures*, Vol. 15, 1990, pp 1-24.

[2] Y. Tseng and Y. Jiang, "Stress Analysis of Bimodulus Laminates Using Hybrid Stress Plate Elements," *Int. J. Solids and Structure*, Vol. 35, 1998, pp. 2025-2038.

[3] K. Khan et al, "Free Vibration of Bimodulus Laminated Cross-ply Conical Panels," *Proc. of the 9th Biennial ASME conference on Engineering System Design and analysis (ESDA'08),* July 7-9, Haifa, Israel

[4] C. W. Bert et al, "Vibration of Thick Rectangular Plates of Bimodulus Composite Material," *ASME Journal of Applied Mechanics,* Vol. 48, 1981, pp. 371-376.

[5] J. L. Doong and L.W. Chen, "Vibration of a Bimodulus Thick Plate," *ASME Journal of Vibration , Acoustics, Stress and Reliability Design*, Vol. 107, 1985, pp. 92-97.

[6] B. P. Patel et al, "Free Flexural Vibration Behavior of Bimodular Material Angle-ply Laminated Composite Plates," *Journal of Sound and Vibration*, Vol. 286, 2005, pp. 167-185.

[7] K. Khan et al, "Vibration Analysis of Bimodulus Laminated Cylindrical Panels," *Journal of Sound and Vibration*, Vol. 321, 2009, pp. 166-183.

[8] K. Khan et al, "Effect of Bimodularity on Dynamic Response of Cross-ply Conical Panels," *Proc. of the International Conference and Exhibition on Aerospace Engineering (ICEAE'09),* May 18-22, Bangalore, India.

[9] K. Khan et al, "Dynamic Analysis of Bimodulus Laminated Angle-ply Conical Panels," *Proc. of the 22nd Canadian Congress of Applied Mechanics (CANCAM'09),* May 31st- 4th June, Halifax, Canada.

[10] K. Khan et al, "Frequency Response of Bimodular Cross-ply Laminated Cylindrical Panels," *Journal of Sound and Vibration, doi:10.1016/j.jsv.2009.05.026*.