Optimal Control of Planar Grasping

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Abstract

Optimal control of planar grasping by multi-finger hand is addressed in this paper. Real time control of manipulation of planar object has to address the force distribution problem which is an under constrained problem and requires optimization of the force system. The proposed control law finds the optimal fingertip force required to manipulate the object through exact optimization of the friction cone angles at the contact point. This guarantees the stability of the grasp without causing slip and consequently loss of contact with the grasped object during manipulation. Demonstration of manipulation of object grasped by two fingers in Matlab is simulated.

Keywords: Grasping, Force distribution, friction angle, Contact Optimization

1 Introduction

Autonomous Robotic Dexterous Manipulation demands skills in mechanism design, force allocation, control theory, and AI strategies. The real time control of grasping requires the determination of suitable grasping forces to balance the external force acting on the grasped object and maintain the grasp stability by optimizing the friction angles at finger contacts. Therefore force distribution involves force optimization. Cheng and Orin [8] and Klein and Kittivatcharapong [9] have solved force optimization problem as a linear programming problem with linearized friction constraints using simplex method. But these techniques are computationally intensive and are unsuitable for real-time implementation. Demmel and Lafferriere [5] reduced the nonlinear friction constraint problem into an eigenvalue problem. This method cannot be extended in closed form to situations where arbitrary equilibrating forces are generated outside the grasp plane. Nakamura [6] proposed nonlinear programming techniques using Lagrange multipliers. This method is time consuming and there is no mention about real-time implementation. Buss [7] expressed the nonlinear friction cone constraints as equivalent positive definiteness of certain symmetric matrix subject to linear constraints and developed gradient flow algorithm for real-time computation. This method is computationally efficient but requires a valid initial grasp force, which satisfies the friction cone constraint to start the gradient algorithm. Mukherjee [3] proposed a closed form solution for optimal interaction force which is slow in implementation as eighth order polynomial is to be solved

Impedance control of position servoed actuators limits the speed at which the finger and object contact force can change, thus inhibiting sudden changes in the constraint structure. Local stability of such grasps can be guaranteed, but not over the complete work space. This problem is skirted when using force control techniques. Chung [1] proposed a computed torque method combined with optimizing the friction angles with arbitrary gain setting. Fuhua [2] proposed force control law using angular rates of object as feedback demonstrating Lyapunov, asymptotic and global asymptotic stability under appropriate conditions.

The specific grasping issue is to maintain the stability of the object during manipulation. The proposed control law finds the optimal fingertip force required to manipulate the object combined with optimizing the friction cone angles at the contact point. This guarantees the stability of the object manipulation without causing slipping and damage to the grasped object.

The contact forces can be decomposed into two parts i.e. equilibrating force which is also called as minimum norm solution which equilibrates the external wrench acting on object and interaction force also known as null solution as it causes no net force to be exerted on the object but is used to modify the contact stability locally.

2 System Dynamics

Dynamic description of the system consisting of a multi-fingered robot hand and an object held by hand has been reported in [1]. The following segment describes dynamics without deriving the equations.

The general motion equations of the object expressed in the base (inertial) frame can be written as:

$$\mathbf{M_o}\ddot{\mathbf{r}}_\mathbf{o} + \mathbf{C}(\mathbf{r_o}, \dot{\mathbf{r}}_\mathbf{o})\dot{\mathbf{r}}_\mathbf{o} = \mathbf{F_o} \qquad (1)$$

Where $\boldsymbol{M_o}$ represent the combined mass and inertia matrix, $\boldsymbol{r_o}$ represent the position and orientation of the object and $\boldsymbol{F_o}$ represents the forces and torques applied to the object at its center of mass.

In planar grasping the object dynamics are somewhat simplified since the object is only allowed to rotate about the axis perpendicular to the plane of motion. If the position and orientation of the object is given by $\boldsymbol{r_o} = (x_o, y_o, \phi_o)$ and the inertia of the object as $I_c \in \mathbb{R}$, it is invariant under rotation, then

$$\begin{bmatrix} m_o & 0 & 0 \\ 0 & m_o & 0 \\ 0 & 0 & I_o \end{bmatrix} \begin{bmatrix} \ddot{x}_o \\ \ddot{y}_o \\ \ddot{\phi}_o \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ \tau_\phi \end{bmatrix} \qquad (2)$$

The dynamics of finger-object contact is described by a mapping between forces exerted by the finger at the point of contact and the force and the moment that can be resisted by the body at some reference point (usually the C.G) on the object.

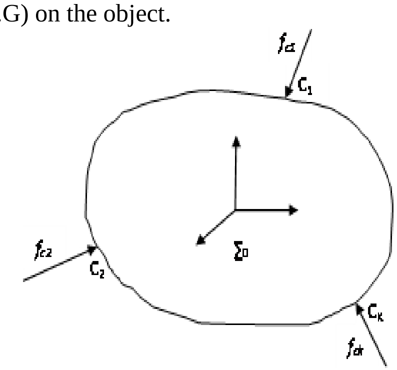

Fig. 1: Rigid Object Grasped by *k*-fingers.

Suppose *k*-fingers are contacting the object as shown in Fig. (1). Point contact with friction is assumed at each contact point. Each contact applies a force $f_{ci} \in \mathbb{R}^3$ through the contact point r_{ci} to the object. The resultant wrench, $F_o = [f_o, \tau_o] \in \mathbb{R}^6$ applied to the object by multi-fingered hands are given by

$$\mathbf{F_o} = \begin{bmatrix} G_1 & \cdots & G_k \end{bmatrix} \begin{bmatrix} f_{c1} \\ \vdots \\ f_{ck} \end{bmatrix} = \mathbf{G}\mathbf{f_c} \qquad (3)$$

Where G_i is a grasp matrix at the contact point C_i given by

$$G_i = \begin{bmatrix} I_3 \\ [r_c \times] \end{bmatrix} \qquad (4)$$

Where $I_n \in \mathbb{R}^{3\times 3}$ is an identity matrix, r_{ci} is the position and orientation vector of i^{th} contact point and $[r_c \times] \in \mathbb{R}^{3\times 3}$ is a skew-symmetric matrix expressing the cross-product from r_{ci}. The grasp matrix G, maps the fingertip forces to the equivalent object wrench. The grasp map is represented as $6 \times n$ matrix, where n is the number of contact forces generated by the fingers. The grasp matrix is calculated based on the position and orientation of the object and the fingertips.

3 Force Distribution

The general formulation for the force distribution has been derived in [1] and [4]. The case of multi-fingered hand manipulating object with hard point contact is presented in this section. To determine the force required for manipulation screw theory is used. Mason and Salisbury [10] used screw theory to characterize the nature of contacts between multi-fingered hands and objects. This formulation is neat and makes the calculations simple.

Force distribution is an inverse dynamic problem i.e. the motion of system is specified and actuation forces/torques to effect this motion is to be determined. In general, the contact force exerted by a multi-fingered hand can be decomposed into two parts, first the equilibrating force, which is determined by the specified manipulation task, and second the internal force, which has no effect on the equilibrium but can be used to modify the contact force to achieve firm fingertip contact.

The fingertip force f_c should be such that it should equilibrate the external wrench acting on the object. For any external wrench F_o acting on the object, the general solution to (4) has the form

$$\mathbf{f_c} = \mathbf{G^+ F_o} + \mathbf{N}\boldsymbol{\lambda} \qquad (5)$$

Where $f_p = G^+ F_o$ is the pseudo-inverse solution and belongs to the row space of G. It is referred to as minimum norm solution which equilibrates the external wrench acting on the object. However to achieve positive normal contact and manage the friction angle, the null solutions have to be superimposed upon the minimum norm solution. These forces have no internal forces.

$f_h = N\lambda$ is the homogeneous solution and belongs to the null space of grasp matrix G, denoted by $N(G)$. Internal forces between two contact points are the component of the differences of the net contact forces along the line joining the two contact points.

The net forces at the n contact points can be of the form

$$\begin{aligned} \mathbf{P_1} &= \mathbf{F_{e_1}} + \sum_{i=2}^{n} \mathbf{k_{1i} u_{1i}} \\ \mathbf{P_2} &= \mathbf{F_{e_2}} + \sum_{i=1}^{n} \mathbf{k_{i2} u_{2i}} \quad \mathbf{i \neq 2} \\ \mathbf{P_r} &= \mathbf{F_{e_r}} + \sum_{i=r}^{n} \mathbf{k_{ir} u_{ri}} \quad \mathbf{i \neq r} \\ \mathbf{P_n} &= \mathbf{F_{e_n}} + \sum_{i=2}^{n-1} \mathbf{k_{in} u_{ni}} \end{aligned} \qquad (6)$$

Where F_i are the net forces and F_{ei} are the equilibrating forces at the contact point. $u_{ij} = (r_{ci} - r_{cj})/|r_{ci} - r_{cj}|$ is the unit direction vector along which interaction forces are to be applied, also $u_{ij} = -u_{ji}$ and $k_{ij} = k_{ji}$ is the associated scalar factor. Consequently, equal and opposite forces are added along the lines of contact between point pairs.

4 Formulation of Min-MaxProblem

The friction angle θ_i at the contact point C_i is

$$\cos\theta_i = \frac{n_i \cdot P_i}{\| P_i \|} \tag{7}$$

Where n_i is the unit normal to the surface at the contact point C_i. The limiting friction problem can be formulated as that of finding a set of internal force such that the angle θ_i does not exceed the maximum allowable friction angle. As a result, the maximum friction angles will be the objective function to be minimized. A grasp situation which satisfies this condition is a stable grasp.

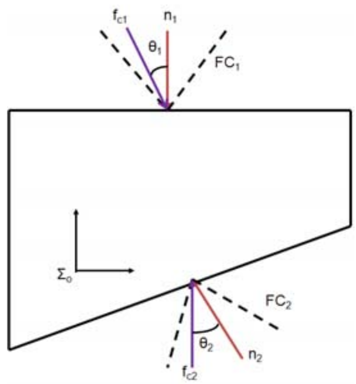

Fig. 2: Object Grasped by *2*-fingers.

For two point frictional contact force vectors can be represented as

$$\begin{aligned} \mathbf{P_1} &= \mathbf{F_{e_1}} + \mathbf{k_{12}u_{12}} \\ \mathbf{P_2} &= \mathbf{F_{e_2}} + \mathbf{k_{21}u_{21}} \end{aligned} \tag{8}$$

Where P_i are the net forces and F_{ei} are the equilibrating forces at the contact point C_i, with i=1, 2. The u_{12} is the unit direction vector along which the interacting force for finger1 is going to act and $u_{21} = -u_{12}$. $k_{12} = k_{21}$ are the associated scalar factor. Correspondingly the friction cone angle θ_1 and θ_2 at the contact point is obtained as

$$\begin{aligned} \cos\theta_1 &= \frac{n_1 \cdot P_1}{\| P_1 \|} \\ \cos\theta_2 &= \frac{n_2 \cdot P_2}{\| P_2 \|} \end{aligned} \tag{9}$$

Where n_1 and n_2 are the unit normal to the surface at the contact point C_1 and C_2. The development below produces a set of equations, solution of which yields, the most stable grasp possible using a set of contact points. This is achieved by maximizing the minimum of the two friction angles for a set of two contact points

4.1 Closed Form Solution of Min-Max

Mukherjee [3] proposed the closed form solution of min-max problem by using cosine function, but its solution requires the evaluation of an 8th order polynomial which is computationally intensive though it is a logical choice. Considering *tan* (θ_i) results in quadratic equation for obtaining the solution which is defined as

$$\tan\theta_i = \frac{\mathbf{r_i} \times \mathbf{P_i}}{\mathbf{n_i} \times \mathbf{P_i}} \tag{10}$$

For two point contact

$$\begin{aligned} \tan\theta_1 &= \frac{\mathbf{r_1} \times \mathbf{F_{e1}} + \mathbf{k_{12}r_1} \times \mathbf{u_{12}}}{\mathbf{n_1} \times \mathbf{F_{e1}} + \mathbf{k_{12}n_1} \times \mathbf{u_{12}}} \\ \tan\theta_2 &= \frac{\mathbf{r_2} \times \mathbf{F_{e2}} + \mathbf{k_{21}r_2} \times \mathbf{u_{21}}}{\mathbf{n_2} \times \mathbf{F_{e2}} + \mathbf{k_{21}n_2} \times \mathbf{u_{21}}} \end{aligned} \tag{11}$$

For solution two cases are considered

$$\begin{aligned} \tan\theta_1 &= \tan\theta_2 \\ \tan\theta_1 &= -\tan\theta_2 \end{aligned} \tag{12}$$

This gives us the quadratic equation in terms of unknown scalar parameter k. That value of k is selected for which the value of *cos(θ)* is maximum since the min-max problem for the θ translates into a max-min problem for the *cos(θ).*

5 Control Algorithm

The goal of control scheme is to determine optimal resultant contact force and optimal internal forces so that the grasped object undergoes a desired motion. The control scheme used is based on linear time invariant optimal control. The model is defined by the state equations is of the form

$$\mathbf{\dot{X}(t) = f(X(t)) + g(X(t))U(t)} \tag{13}$$

Linearizing the state equation at time t_1 the state equation is reduced to form

$$\mathbf{\dot{X}(t) = A(t_1)X(t) + B(t_1)U(t)} \tag{14}$$

Where X is nx1, A is nxn, B is nxp and U is Px1. The control scheme used is based on linear time invariant optimal control. The linear quadratic control law for Eq. (14), where *(A,B)* is completely controllable is given by

$$\mathbf{U(t) = -K(t)X(t)} \tag{15}$$

Where $K \in p \times n$ is a real, constant and unconstrained gain matrix, that minimizes the following performance function which is sum of norm of states and norm the input force acting at each contact point subject to initial conditions $X(0)=X^0$:

$$J = \int_0^\infty (\mathbf{X^T(t)QX(t) + U^T(t)RU(t)})dt \tag{16}$$

Where the matrices $Q \in n \times n$ and $R \in p \times p$ are positive semi-definite and positive-definite, respectively. The solution is obtained by solving the matrix Ricatti equation given by

$$\mathbf{A^TP + PA - K^TB^TP - PBK + K^TRK = -Q} \tag{17}$$

The control gain matrix that minimizes the cost function is given by:

$$\mathbf{K(t) = R^{-1}B^TP(t)} \tag{18}$$

The block diagram is shown in Fig. (2). with force optimzation. The optimal control problem defined as the regulator problem with online force optimization. The LQR controller is designed to drive the states to equilibrium position in a manner that it optimizes some performance measure.

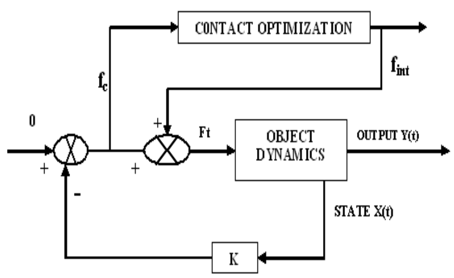

Fig. 3: Regulator Problem with Contact Optimization

The object dynamics is modified to incorporate environment force

$$\mathbf{M_o \ddot{X}_o + K_s (X_o - X_r) = F_o = G f_c} \qquad (19)$$

Where $K_s = (k_x, k_y, k$ is an arbitrary, locally linearized, stiffness applied by the environment on the object.

6 Simulation

In order to show the effectives of proposed algorithm, a numerical example is simulated. An planar object grasped by two fingers with the assumption that object is rigid and finger-object interaction is through a point contact with friction. The reference frame to start with has been assumed to be at the center of mass of the object.

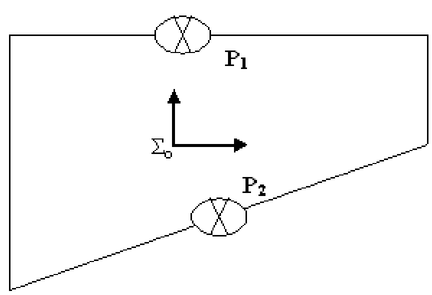

Fig. 4: Planar Object Grasped by 2-fingers

The Fig. (5). shows the object grasped by two fingers. In this formulation, the origin of coordinate system is at the C.G of the object. Consider a planar object contacted at two opposite faces. The points of contact on object in local coordinates are r_{cx1}= (0.05, 0.07, 0) and r_{cx2}= (0.05, 0.0289, 0), mass of object, m_o=0.05 kg and moment of inertia of object, $I_o=1.3202x10^{-5} kg$-m^2. The state space formulation is

$$\mathbf{\dot{X}(t) = A(t)X(t) + B(t)U(t)} \qquad (20)$$

Where ***X(t)*** is *6*x1, ***A*** is *6*x*6*, ***B*** is *6*x*4* and ***U*** is *4*x1. The value of spring stiffness is taken as $k_x = 15\ N/m$, $k_y = 12\ N/m$ and $k_\phi = 0.3881\ Nm/rad$. The A matrix and B matrix are of form

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ K_x/m_o & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & K_y/m_o & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & K_{phi}/I_o & 0 \end{bmatrix} \qquad (21)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1/m_o & 0 & 1/m_o & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1/m_o & 0 & 1/m_o \\ 0 & 0 & 0 & 0 \\ -p_{y1}/I_o & -p_{x1}/I_o & -p_{y2}/I_o & p_{x2}/I_o \end{bmatrix}$$

(22)

$$U^T = \begin{bmatrix} f_{cx1} & f_{cy1} & f_{cx2} & f_{cy2} \end{bmatrix};$$

$$X^T(t) = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}$$

(23)

The object is regulated to equilibrium position from the current position of the object. The weighting matrices are user specified and define the trade-off between regulation performance (how fast goes to zero) and control effort. The LQR regulator Gain matrix K_{lqr} is computed with the following weighting matrices

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(24)

The force required to reposition the object is computed. The contact optimization is done to check the stability of the object.

7 Result

In order to show the effectives of proposed algorithm, a numerical example is simulated. An planar object grasped by two fingers with the assumption that object is rigid and finger-object interaction is through a point contact with friction. The reference frame to start with has been assumed to be at the center of mass of the object.

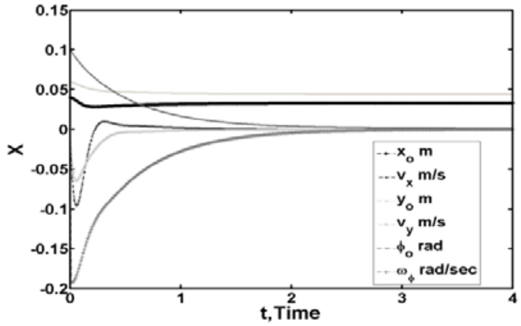

Fig. 5: System States

The time trace of states posed as regulator problem during manipulation is shown in Fig. (5). The system states are driven to equilibrium point x=0.3345, y=0.4456, which is the C.G of the object in 2 sec.

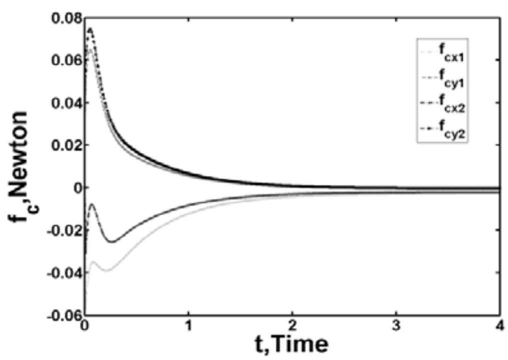

Fig. 6: Fingertip force without internal force

The equilibrating force is non-zero at steady state as the environmental force has to be compensated shown in Fig. (6).

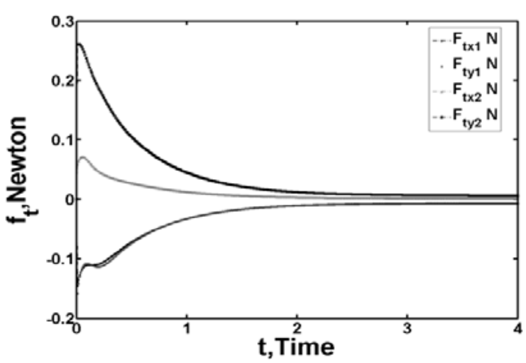

Fig. 7: Total Fingertip force with internal force

The fingertip force is modified by internal force is shown in Fig. (7). This force will hold object stably during manipulation satisfying friction cone constraint.

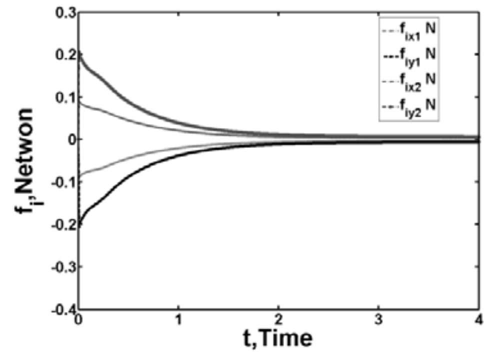

Fig. 8: Internal force

Fig. (8). shows the interaction force which is used to modify the equilibrating forces to hold the object stably.

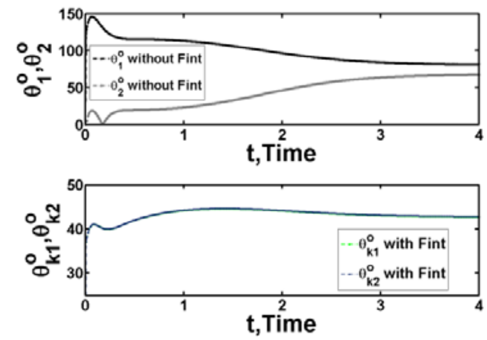

Fig. 9: Friction angle without and with Contact Optimization

Fig. (9). shows the friction angle with and without internal force. The infeasible solution where the angle is greater than 90^o is modified by the contact optimization and satisfies the feasible solution. The friction angle should be equal is satisfied as demanded by the algorithm and is 43^o.

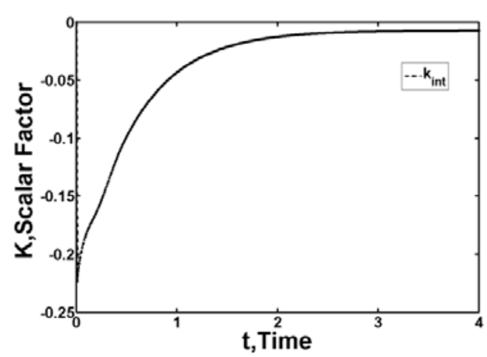

Fig. 10: Scalar Parameter

Fig. (10). shows the scalar parameter k peaks the value of -0.223 before settling down to value of -0.0074.

8 Conclusion

Solution to the regulator problem in 2D grasp situation has been demonstrated through a 2 finger example. The contact force optimization leads to physically realizable solution that is not suggested by the solution through LQR formulation. The methodology is to be extended to three fingered and 3-D grasp situation.

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